### **Review of Main Concepts (Counting)**

- Sum Rule: If an experiment can either end up being one of N outcomes, or one of M outcomes (where there is no overlap), then the total number of possible outcomes is N + M.
- **Product Rule:** Suppose events  $A_1, ..., A_n$  each have  $m_1, ..., m_n$  possible outcomes, respectively. Then there are  $m_1 \cdot m_2 \cdot m_3 \cdots m_n = \prod_{i=1}^n m_i$  possible outcomes overall.
- Number of ways to order n distinct objects:  $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$
- Number of ways to select from n distinct objects:
  - Permutations (number of ways to linearly arrange k objects out of n distinct objects, when the order of the k objects matters):

$$P(n,k) = \frac{n!}{(n-k)!}$$

Combinations (number of ways to choose k objects out of n distinct objects, when the order of the k objects does not matter):

$$\frac{n!}{k!(n-k)!} = \binom{n}{k} = C(n,k)$$

#### 1. Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if ...

- (a) ...all couples are to get adjacent seats?
- (b) ...anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

## 2. Weird Card Game

In how many ways can a pack of fifty-two cards be dealt to thirteen players, four to each, so that every player has one card of each suit?

#### 3. HBCDEFGA

How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

#### 4. Escape the Professor

There are 6 security professors and 7 theory professors taking part in an escape room. If 4 security professors and 4 theory professors are chosen and paired off, how many pairings are possible?

## 5. Birthday Cake

A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

## 6. Full Class

There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

# 7. Paired Finals

Suppose you are to take a CSE 312 final in pairs. There are 100 students in the class and 8 TAs, so 8 lucky students will get to pair up with a TA. Each TA must take the exam with some student, but two TAs cannot take the exam together. How many ways can they pair up?

## 8. Photographs

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

## 9. Rabbits!

Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

## 10. Extended Family Portrait

A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?

### 11. Subsubset

Let  $[n] = \{1, 2, ..., n\}$  denote the first *n* natural numbers. How many (ordered) pairs of subsets (A, B) are there such that  $A \subseteq B \subseteq [n]$ ?

### 12. Divide Me

How many numbers in [360] are divisible by:

- (a) 4, 6, and 9?
- (b) 4, 6, or 9?
- (c) Neither 4, 6, nor 9?