CSE 312 Section 1

Combinatorics

Administrivia & Introductions

Announcements & Reminders

- Section Materials
  - Handouts will be provided in at each section
  - Worksheets and sample solutions will be available on the course calendar later this evening
- Office Hours
  - We start holding office hours today!
  - Times posted on the calendar on the course website
- HW1
  - Due Wednesday 4/3 @ 11:59pm

Homework

- Submissions
    - LaTeX (highly encouraged)
        - overleaf.com
        - template and LaTeX guide posted on course website!
    - Word Editor that supports mathematical equations
    - Handwritten neatly and scanned
- Homework will typically be due on Wednesdays at 11:59pm on Gradescope
- Each assignment can be submitted a max of 72 hours late
- You have **8 late days total** to use throughout the quarter
    - Anything beyond that will result in a 15% deduction per day

Icebreaker

- Small groups of 4-6ish
- Please share with your group
  - Your name
  - Number of years in department/ at UW
  - What was something fun you did over Spring break?
  - What are you concerned about for 312 / what are you excited about?
- Then, share how you like to eat your potatoes (baked, fried, chips, etc)
- We'll go around and see what style of potato is most popular!

Review

Any lingering questions from this last week?

Each week in section, we’ll be reviewing the main concepts from this week by going through some practice problems together. But before that, we’ll try to start off each section with some time for you to ask questions.

Was anything particularly confusing this week? Is there anything we can clarify before we dive into the review? This is your chance to clear things up!

Problem 1 - Seating

1 – Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if …

a) … all couples are to get adjacent seats?
b) … anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

Work on this problem with the people around you, and then we'll go over it together!

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Consider each couple as a unit.

Apply the product rule, first choosing one of the 5! permutations of the 5 couples, and then, for each couple in turn, choosing one of the 2 permutations for how they sit (for a total of 2^5). Therefore, the answer is:

$$5! \cdot 2^5$$

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One way we can solve this is to consider by cases.

Name the two people in the couple A and B. There are two cases: A can sit on one of the ends, or not.

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One way we can solve this is to consider by cases.

Name the two people in the couple A and B. There are two cases: A can sit on one of the ends, or not.
Case 1: If A sits on an end seat, A has 2 choices and B has 8 possible seats.
Case 2: If A doesn't sit on the end, A has 8 choices and B only has 7.
So, there are a total of $2 \cdot 8 + 8 \cdot 7$ ways A and B can sit.

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So, there are a total of $2 \cdot 8 + 8 \cdot 7$ ways A and B can sit.

Once they do, the other 8 people can sit in 8! ways since there are no other restrictions.
Hence the total number of ways is

$$(2 \cdot 8 + 8 \cdot 7)8! = 9 \cdot 8 \cdot 8! = 8 \cdot 9!$$

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b) How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

Another way to solve this is to apply complementary counting to first compute the total number of arrangements of the 10 people, and then subtract from this the number of arrangements in which that particular couple does get adjacent seats.

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Ways for a particular couple to get adjacent seats. Since you can treat the couple as a unit, permute the 9 “individuals” (consisting of 8 people plus the couple) and then consider the 2 permutations for that couple: $9! \cdot 2$

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Total arrangements: 10!
Ways for a particular couple to get adjacent seats. Since you can treat the couple as a unit, permute the 9 "individuals" (consisting of 8 people plus the couple) and then consider the 2 permutations for that couple: $9! \cdot 2$

That means the answer to the question is

$$10! - 9! \cdot 2 = 9!\,(10 - 2) = 8 \cdot 9!$$

Problem 5 – Birthday Cake

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A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

Work on this problem with the people around you, and then we'll go over it together!

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Given the 1 choice on Thursday, for each of Wednesday and Friday, there are 4 choices (the different pie options).

Given the choice on Wednesday, there are 4 choices for Tuesday, and given the choice on Tuesday, there are 4 choices for Monday, and given the choice on Monday, there are 4 choices on Sunday. Similarly, given the choice on Friday, there are 4 choices on Saturday.

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Therefore the answer is

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 1 \cdot 4 \cdot 4 = 4^6$$

Problem 10 – Extended Family Portrait

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A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?

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Then consider each of the n families one by one and reorder their members. Within each family, there are $m!$ ways to order their members.

So, the total number of ways to line these people up according to the given constraints is

$$n!\,(m!)^n$$

Problem 9 – Rabbits!

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Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

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Find the number of possibilities when Peter and Pauline go to the same store and find the number of possibilities when they go to different stores, and then use the sum rule to get the final answer.

If Peter and Pauline go to the same store, there are 4 stores it could be. For each such choice, there are 3 choices of store for each of the 3 offspring, so 3^3 choices for all the offspring. If Peter and Pauline go to different stores, there are $4 \cdot 3 = 12$ pairs of stores they could go to. For each such choice, there are 2 choices of store for each of the 3 offspring, so 2^3 choices for all the offspring. Therefore the answer is

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$$4 \cdot 3^3 + 12 \cdot 2^3$$

Problem 4 – Escape the Professor

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There are 6 security professors and 7 theory professors taking part in an escape room. If 4 security professors and 4 theory professors are chosen and paired off, how many pairings are possible?

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The answer is

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The answer is

$$\binom{6}{4}\binom{7}{4}4!$$

Problem 11 - Subsubset

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Let $[n] = \{1,2, \dots, n\}$ denote the first n natural numbers. How many (ordered) pairs of subsets (A, B) are there such that $A \subseteq B \subseteq [n]$?

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Realize that, if there are no restrictions, for each element i of $1, \ldots, n$, there are four possibilities: it can be in only A, only B, both, or neither. In our case, there is only one that is not valid (violates $A \subseteq B$): being in A but not B. Hence there are 3 choices for each element, so the total number of such ordered pairs of subsets is

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$$3^n$$

That’s All, Folks!

Thanks for coming to section this week!
Any questions?