Preliminary: Random Vectors

A random vector X is a vector where each entry is a random variable.

$\mathbb{E}[X]$ is a vector, where each entry is the expectation of that entry.

For example, if X is a uniform vector from the sample space

$$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-1\\2\\3\end{bmatrix}, \begin{bmatrix}0\\2\\6\end{bmatrix}\right\}$$

$\mathbb{E}[X] = [0,2,4]^T$

Covariance Matrix

Remember Covariance?

$\text{Cov}(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$

We'll want to talk about covariance between entries:

Define the "covariance matrix"

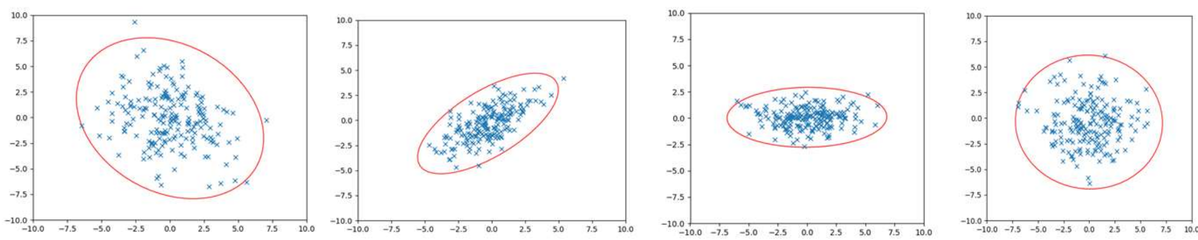
$$\Sigma = \begin{bmatrix} \text{Cov}(X_1, X_1) & \cdots & \text{Cov}(X_1, X_n) \\ \vdots & \text{Cov}(X_i, X_j) & \vdots \\ \text{Cov}(X_n, X_1) & \cdots & \text{Cov}(X_n, X_n) \end{bmatrix}$$

Dependence

Let's think about 2 dimensions.

Let $X = [X_1, X_2]^T$ where $\mathrm{Var}(X_1) = 3$, $\mathrm{Var}(X_2) = 3$ BUT X_1 and X_2 are dependent. $\mathrm{Cov}(X_1, X_2) = 2$

What is Σ? Which of these pictures are i.i.d. samples of X?



Practice with conditional expectations

Consider of the following process:

Flip a fair coin, if it's heads, pick up a 4-sided die; if it's tails, pick up a 6-sided die (both fair)

Roll that die independently 3 times. Let X_1, X_2, X_3 be the results of the three rolls.

What is $\mathbb{E}[X_2]$? $\mathbb{E}[X_2|X_1 = 5]$? $\mathbb{E}[X_2|X_3 = 1]$?