Frogs

There are 20 frogs on each location in a 5x5 grid. Each frog will independently jump to the left, right, up, down, or stay where it is with equal probability. A frog jumping off an edge of the grid magically warps to the opposite edge (pac-man-style).

Bound the probability that at least one square ends up with at least 36 frogs.

These events are dependent – adjacent squares affect each other!

Union Bound

For any events E, F

$$\mathbb{P}(\boldsymbol{E} \cup \boldsymbol{F}) \leq \mathbb{P}(\boldsymbol{E}) + \mathbb{P}(\boldsymbol{F})$$

(Multiplicative) Chernoff Bound

Let $X_1, X_2, \ldots, X_n$ be *independent* Bernoulli random variables.
***Let* $X = \sum X_i$*,* and $\mu = \mathbb{E}[X]$. For any $0 \leq \delta \leq 1$**

$$\mathbb{P}(X \geq (1+\delta)\mu) \leq \exp\left(-\frac{\delta^2\mu}{3}\right) \textbf{ and } \mathbb{P}(X \leq (1-\delta)\mu) \leq \exp\left(-\frac{\delta^2\mu}{2}\right)$$

Doing Better With Randomness

You don't really need to know who was cheating. Just how many people were.

Here's a protocol:

Please flip a coin.

If the coin is heads, or you have ever cheated, please tell me "heads"

If the coin is tails and you have not ever cheated, please tell me "tails"

Hoeffding's Inequality

Hoeffding's Inequality

Let $X_1, X_2, \ldots, X_n$ be *independent* RVs, each with range $[0,1]$.
Let **$\bar{X} = \sum X_i / n$, and $\mu = \mathbb{E}[\bar{X}]$. For any $t \geq 0$**

$$\mathbb{P}(|\bar{X} - \mathbb{E}[\bar{X}]| \geq t) \leq 2\exp(-2nt^2)$$

How close will we be with *n=1000 with probability at least* .95*?*

$|X - \mathbb{E}[X]| \geq t$ if and only if $|Y - \mathbb{E}[Y]| \geq 2t$.