Near the mean

Suppose you run a poll of 1000 people where in the true population 60% of the population supports you. What is the probability that the poll is not within 10-percentage-points of the true value?

$\bar{X} = \sum X_i/1000$

$\mathbb{E}[\bar{X}]$

$\text{Var}(\bar{X})$

Chebyshev's Inequality

Let X be a random variable. For any $t > 0$

$$\mathbb{P}(|X - \mathbb{E}[X]| \geq t) \leq \frac{\text{Var}(X)}{t^2}$$

More Assumptions → Better Guarantee

(Multiplicative) Chernoff Bound

Let $X_1, X_2, \ldots, X_n$ be *independent* Bernoulli random variables.
***Let* $X = \sum X_i$, and $\mu = \mathbb{E}[X]$. For any $0 \leq \delta \leq 1$**

$$\mathbb{P}(X \geq (1+\delta)\mu) \leq \exp\left(-\frac{\delta^2\mu}{3}\right) \text{ and } \mathbb{P}(X \leq (1-\delta)\mu) \leq \exp\left(-\frac{\delta^2\mu}{2}\right)$$

Left Tail

Suppose you run a poll of 1000 people where in the true population 60% of the population supports you. What is the probability that the poll is not within 10-percentage-points of the true value?

Want $\mathbb{P}\left(\frac{X}{1000} \leq .5\right) = \mathbb{P}(X \leq .5 \cdot 1000)$

Chernoff Bound (left tail)
Let $X_1, X_2, \ldots, X_n$ be *independent* Bernoulli random variables. *Let* $X = \sum X_i$, and $\mu = \mathbb{E}[X]$. For any $0 \leq \delta \leq 1$ $\mathbb{P}(X \leq (1-\delta)\mu) \leq \exp\left(-\frac{\delta^2 \mu}{2}\right)$

One More Bound

The Union bound

Union Bound
For any events E, F $\mathbb{P}(\boldsymbol{E} \cup \boldsymbol{F}) \leq \mathbb{P}(\boldsymbol{E}) + \mathbb{P}(\boldsymbol{F})$

Proof? $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$

And $\mathbb{P}(E \cap F) \geq 0$.