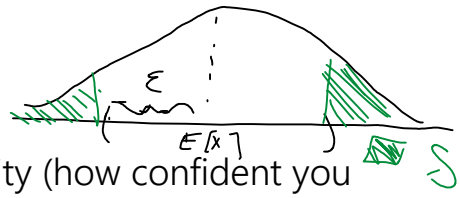
Confidence Intervals



A "confidence interval" tells you the probability (how confident you should be) that your random variable fell in a certain range (interval)

Usually "close to its expected value"

$$\mathbb{P}(|X - \mu| > \varepsilon) \leq \delta$$

equivalently $\mathbb{P}(|X - \mu| \leq \varepsilon) > 1 - \delta$

If your RV has expectation equal to the value you're searching for (like our polling example) you get a probability of being "close enough" to the target value.

A Second Example

Suppose the average number of ads you see on a website is 25. Give an upper bound on the probability of seeing a website with 75 or more ads.

Markov's Inequality

Let X be a random variable supported (only) on non-negative numbers. For any $t > 0$

$$\mathbb{P}(X \geq t) \leq \frac{\mathbb{E}[X]}{t}$$

Our First Two Inequalities

Markov's Inequality

Let X be a random variable supported (only) on non-negative numbers. For any $t > 0$

$$\mathbb{P}(X \geq t) \leq \frac{\mathbb{E}[X]}{t}$$

Chebyshev's Inequality

Let X be a random variable. For any $t > 0$

$$\mathbb{P}(|X - \mathbb{E}[X]| \geq t) \leq \frac{\mathbf{Var}(X)}{t^2}$$

Chebyshev's – Repeated Experiments

How many coin flips (each head with probability p) are needed until you get n heads.

Let X be the number necessary. What is probability $X \geq 2n/p$?

Markov

Chebyshev