Let's start with the pmf

For discrete random variables, we defined the pmf: $p_Y(k) = \mathbb{P}(Y = k)$.

We can't have a pmf quite like we did for discrete random variables. Let X be a random real number between 0 and 1.

$\mathbb{P}(X = .1) = \frac{1}{\infty}$??

Let's try to maintain as many rules as we can…

Continuous	Discrete
$p_Y(k) \geq 0$	$f_X(k) \geq 0$
$\sum_{\omega} p_Y(\omega) = 1$	$\int_{-\infty}^{\infty} f_X(k)\ \mathrm{d}k$

Use f_X instead of p_X to remember it's different .

The probability density function

For Continuous random variables, the analogous object is the "probability density function" we write $f_X(k)$ instead of $p_X(k)$

Idea: Make it "work right" for **events** since single outcomes don't make sense.

$\mathbb{P}(0 \leq X \leq 1) = 1$

integrating is analogous to sum.

$\mathbb{P}(X \text{ is negative}) = 0$

$\mathbb{P}(.4 \leq X \leq .5) = .1$

What about $\mathbb{E}[g(X)]$

Let $X \sim \text{Unif}(a, b)$, what about $\mathbb{E}[X^2]$?

$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} z^2 f_X(z) \mathrm{d}z$

$=$

Comparing Discrete and Continuous

	Discrete Random Variables	Continuous Random Variables
Probability 0	Equivalent to impossible	All impossible events have probability 0, but not conversely.
Relative Chances	PMF: $p_X(k) = \mathbb{P}(X = k)$	PDF $f_X(k)$ gives chances relative to $f_X(k')$
Events	Sum over PMF to get probability	Integrate PDF to get probability
Convert from CDF to PMF	Sum up PMF to get CDF. Look for "breakpoints" in CDF to get PMF.	Integrate PDF to get CDF. Differentiate CDF to get PDF.
$\mathbb{E}[X]$	$\sum_{\omega} X(\omega) \cdot f_X(\omega)$	$\int_{-\infty}^{\infty} z \cdot f_X(z)\, \mathrm{d}z$
$\mathbb{E}[g(X)]$	$\sum_{\omega} g(X(\omega)) \cdot f_X(\omega)$	$\int_{-\infty}^{\infty} g(z) \cdot f_X(z)\, \mathrm{d}z$
$\text{Var}(X)$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2$	$\mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \int_{-\infty}^{\infty} (z - \mathbb{E}[X])^2 f_X(z)\, \mathrm{d}z$