Where are we?

A random variable is a numerical summary of the outcome of an experiment.

$\mathbb{E}[X]$ is the weighted average of possibilities of X.

For <u>all</u> rv's $\mathbb{E}[X+Y] = \mathbb{E}[X] = \mathbb{E}[Y]$.

Indicator rv's are a great trick to simplify expectation computations.

$\text{Var}(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$ measures how "spread out" a rv is.

For <u>independent</u> rv's:

$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$

Make a prediction

How should $\text{Var}(X+c)$ relate to $Var(X)$ if c is a constant?

How should $\text{Var}(\text{aX})$ relate to $\text{Var}(X)$ is a is a constant?

Zoo!

$X \sim \text{Unif}(a, b)$	$X \sim \text{Ber}(p)$	$X \sim \text{Bin}(n, p)$	$X \sim \text{Geo}(p)$
$p_X(k) = \frac{1}{b-a+1}$ $\mathbb{E}[X] = \frac{a+b}{2}$ $\text{Var}(X) = \frac{(b-a)(b-a+2)}{12}$	$p_X(0) = 1-p;$ $p_X(1) = p$ $\mathbb{E}[X] = p$ $\text{Var}(X) = p(1-p)$	$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$ $\mathbb{E}[X] = np$ $\text{Var}(X) = np(1-p)$	$p_X(k) = (1-p)^{k-1} p$ $\mathbb{E}[X] = \frac{1}{p}$ $\text{Var}(X) = \frac{1-p}{p^2}$

$X \sim \text{NegBin}(r, p)$	$X \sim \text{HypGeo}(N, K, n)$	$X \sim \text{Poi}(\lambda)$
$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$ $\mathbb{E}[X] = \frac{r}{p}$ $\text{Var}(X) = \frac{r(1-p)}{p^2}$	$p_X(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$ $\mathbb{E}[X] = n\frac{K}{N}$ $\text{Var}(X) = \frac{K(N-K)(N-n)}{N^2(N-1)}$	$p_X(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $\mathbb{E}[X] = \lambda$ $\text{Var}(X) = \lambda$

Formally…

Let X be the number of flips needed, Y be the flips after the third.

$\mathbb{P}(Y = k | X \geq 3) = ?$

…

Which is $p_X(k)$.