Linearity of Expectation

For any two random variables X and Y:

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

Note: X and Y do not have to be independent

Repeated Coin Tosses

Now what if the probability of flipping a head was p and that we wanted to find the total number of heads flipped when we flip the coin n times?

Let X be the r.v. representing the total number of heads.

Make a prediction --- what should $\mathbb{E}[X]$ be?

Computing complicated expectations

We often use these three steps to solve complicated expectations

1. Decompose: Finding the right way to decompose the random variable into sum of simple random variables
$$X = X_1 + X_2 + \cdots + X_n$$
2. LOE: Apply Linearity of Expectation
$$\mathbb{E}[X] = \mathbb{E}[X_1] + \mathbb{E}[X_2] + \cdots + \mathbb{E}[X_n]$$
3. Conquer: Compute the expectation of each X_i

Often X_i are indicator random variables

Pairs with the same birthday

In a class of m students, on average how many pairs of people have the same birthday?

<u>Decompose:</u>

<u>LOE:</u>

<u>Conquer:</u>

Rotating the table

n people are sitting around a circular table. There is a name tag in each place. Nobody is sitting in front of their own name tag.

Rotate the table by a random number k of positions between 1 and n-1 (equally likely)

Let X be the number of people that end up in front of their own name tag. Find $\mathbb{E}[X]$.

<u>Decompose:</u>

What X_i can we define that have the needed information?

<u>LOE:</u>

<u>Conquer:</u>