Independence of Random Variables

That's for events…what about random variables?

Independence (of random variables)

X **and** Y **are independent if for all** k, ℓ

$$\mathbb{P}(X = k, Y = \ell) = \mathbb{P}(X = k)\mathbb{P}(Y = \ell)$$

We'll often use commas instead of ∩ symbol.

What does Independence give you?

$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

What does XY mean? I rolled two dice, let X be the red die, Y the blue die. XY is the random variable that tells you the product of the two dice.

That's a function that takes in an outcome and gives you a number back…so a random variable!! (Same for $X + Y$).

Expectations of functions of random variables

Let's say we have a random variable X and a function g. What is $\mathbb{E}[g(X)]$?

$\mathbb{E}[g(X)] = \sum_{\omega \in \Omega} g\big(X(\omega)\big) \cdot \mathbb{P}(\omega)$

Equivalently: $\mathbb{E}[g(X)] = \sum_{k \in \Omega_{\mathrm{g(X)}}} k \cdot \mathbb{P}(g(X) = k)$

Notice that $\mathbb{E}[g(X)]$ might not be $g(\mathbb{E}[X])$.

Variance

Variance

The variance of a random variable X is

$$\mathrm{Var}(X) = \sum_{\omega} \mathbb{P}(\omega) \cdot (X(\omega) - \mathbb{E}[X])^2 = \mathbb{E}\big[(X - \mathbb{E}[X])^2\big] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

The first two forms are the definition. The last one is an algebra trick.