Independence

Independence

Two events A, B are independent if
$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

You'll sometimes see this called "statistical independence" to emphasize that we're talking about probabilities (not, say, physical interactions).

If A, B both have non-zero probability then

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \Leftrightarrow \mathbb{P}(A|B) = \mathbb{P}(A) \Leftrightarrow \mathbb{P}(B|A) = \mathbb{P}(B)$$

Mutual Exclusion and independence

Two of these statements are true, one is false. Explain to each other which ones are true, and find a counter-example to the false one.

1. If A, B both have nonzero probability and they are mutually exclusive, then they cannot be independent.

2. If A has zero probability, then A, B are independent (for any B).

3. If two events are independent, then at least one has nonzero probability.

Fill out the poll everywhere so Robbie knows how long to explain
Go to pollev.com/robbie

Chain Rule

We defined conditional probability as: $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

Which means $\mathbb{P}(A \cap B) = \mathbb{P}(A|B)\mathbb{P}(B)$

Chain Rule

$$\mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n)$$
$$= \mathbb{P}(A_n|A_1 \cap \cdots \cap A_{n-1}) \cdot \mathbb{P}(A_{n-1}|A_1 \cap \cdots \cap A_{n-2}) \cdots \mathbb{P}(A_2|A_1) \cdot \mathbb{P}(A_1)$$

Conditional Independence Example

You have two coins. Coin A is fair, coin B comes up heads with probability 0.85.

You will roll a (fair) die, if the result is odd flip coin A twice (independently); if the result is even flip coin B twice (independently)

Let C_1 be the event "the first flip is heads", C_2 be the event "the second flip is heads", O be the event "the die was odd"

Are C_1 and C_2 independent? Are they independent conditioned on O?