Another Example

Suppose you shuffle a deck of cards so any arrangement is equally likely. What is the probability that the top two cards have the same value?

Sample Space

Probability Measure

Event

Probability

Conditional Probability

Conditional Probability

For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on $\boldsymbol{B}$" is

$$\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$$

Just like with the formal definition of probability, this is pretty abstract.

It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has not happened) – $\mathbb{P}(A|B)$ is **undefined** when $\mathbb{P}(B) = 0$.

Direction Matters

No! $\mathbb{P}(A|B)$ and $\mathbb{P}(B|A)$ are different quantities.

$\mathbb{P}$("traffic on the highway" | "it's snowing") is close to 1

$\mathbb{P}$("it's snowing" | "traffic on the highway") is much smaller; there many other

It's a lot like implications – order can matter a lot!

(but there are some A, B where the conditioning doesn't make a difference)

Conditioning Practice

Red die 6 conditioned on sum 7

Red die 6 conditioned on sum 9

Sum 7 conditioned on red die 6

Take a few minutes to work on this with the people around you! (also on your handout)

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
D1=1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1.6)
D1=2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
D1=3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
D1=4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
D1=5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
D1=6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Let **A** be the event that the red die is 6
Let **B** be the event the sum is 7
Let **C** be the event the sum is 9

1. Find the probability of red die being 6 conditioned on sum 7. This is **P(__ | __) =**

2. Find the probability of red die being 6 conditioned on sum 9. This is **P(__ | __) =**

3. Find the probability of red die being 6 conditioned on sum 7. This is **P(__ | __) =**

Conditioning

Let A be the event the scale ALERTS you

Let B be the event your bar has a ticket.

What probabilities are each of these?

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.

If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

If you pick up a bar and it alerts, what is the probability you have a golden ticket?

Bayes Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

What do we know about Wonka Bars?

$$0.999 = \frac{\mathbb{P}(B|A) \cdot \mathbb{P}(A)}{.001}$$

Filling In

What's $\mathbb{P}(A)$?

We'll use a trick called "the law of total probability":

$$\mathbb{P}(A) = \mathbb{P}(A|B) \cdot \mathbb{P}(B) + \mathbb{P}(A|\bar{B}) \cdot P(\bar{B})$$
$$= 0.999 \cdot .001 + .01 \cdot .999$$
$$= .010989$$

Law of Total Probability

Law of Total Probability

Let $A_1, A_2, \ldots, A_k$ be a **partition** of Ω.
For any event E,

$$\mathbb{P}(\boldsymbol{E}) = \sum_{\text{all } i} \mathbb{P}(\boldsymbol{E}|\boldsymbol{A_i})\mathbb{P}(\boldsymbol{A_i})$$