Probability Space

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A (discrete) probability space is a pair $(\Omega, \mathbb{P})$ where:
Ω is the sample space
$\mathbb{P}: \Omega \to [0,1]$ is the probability measure.
$\mathbb{P}$ satisfies:

- $\mathbb{P}(x) \geq 0$ **for all** x
- $\sum_{x \in \Omega} \mathbb{P}(x) = 1$
- **If** $E, F \subseteq \Omega$ **and** $E \cap F = \emptyset$ **then** $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$

Uniform Probability Space

The most common probability measure is the **uniform** probability measure. In the uniform measure, for every event E

$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$.

Let your sample space be all possible outcomes of a sequence of 100 coin tosses. Assign the uniform measure to this sample space. What is the probability of the event “there are exactly 50 heads?

A. $\binom{100}{50}/2^{100}$

B. $1/101$

C. $1/2$

D. $1/2^{50}$

E. There is not enough information in this problem.

Axioms and Consequences

We wrote down 3 requirements (axioms) on probability measures

- $\mathbb{P}(x) \geq 0$ for all x (non-negativity)
- $\sum_{x \in \Omega} \mathbb{P}(x) = 1$ (normalization)
- If E and F are mutually exclusive then $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$ (countable additivity)

These lead quickly to these three corollaries

- $\mathbb{P}(\bar{E}) = 1 - \mathbb{P}(E)$ (complementation)
- If $E \subseteq F$, then $\mathbb{P}(E) \leq \mathbb{P}(F)$ (monotonicity)
- $\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$ (inclusion-exclusion)

Another Example

Suppose you shuffle a deck of cards so any arrangement is equally likely. What is the probability that the top two cards have the same value?

Sample Space

Probability Measure

Event

Probability