Some Facts about combinations

Symmetry of combinations: $\binom{n}{k} = \binom{n}{n-k}$

Pascal's Rule: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Binomial Theorem

In high school you probably memorized

$(x + y)^2 = x^2 + 2xy + y$^2

And $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

The Binomial Theorem tells us what happens for every n:

The Binomial Theorem

$$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

In general:

$|A_1 \cup A_2 \cup \cdots \cup A_n| =$

$|A_1| + |A_2| + \cdots + |A_n|$

$-(\,|A_1 \cap A_2| + |A_1 \cap A_3| + \cdots + |A_1 \cap A_n| + |A_2 \cap A_3| + \cdots + |A_{n-1} \cap A_n|)$

$+(\,|A_1 \cap A_2 \cap A_3| + \cdots + |A_{n-2} \cap A_{n-1} \cap A_n|)$

$-\ \ldots$

$+(-1)^{n+1}|A_1 \cap A_2 \cap \cdots \cap A_n|$

Add the individual sets, subtract all pairwise intersections, add all three-wise intersections, subtract all four-wise intersections,..., [add/subtract] the n-wise intersection.

Strong Pigeonhole Principle

If you have n pigeons and k pigeonholes, then there is at least one pigeonhole that has at least $\left\lceil \frac{n}{k} \right\rceil$ pigeons.

$\lceil a \rceil$ is the "ceiling" of a (it means always round up, $\lceil 1.1 \rceil = 2$, $\lceil 1 \rceil = 1$).