

# Here Early?

Here for CSE 312?

Welcome! You're early!

handouts up front  
if you want paper  
copy!

Want a copy of these slides to take notes?

You can download them from the webpage [cs.uw.edu/312](http://cs.uw.edu/312)

Want to be ready for the end of the lecture?

Download the Activity slide from the same place

Go to [pollev.com/robbie](http://pollev.com/robbie) and login with your at-uw email (not at-cs!)

You should see a question that doesn't make sense yet.

# Introduction and Counting

CSE 312 24Sp  
Lecture 1

# Staff



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Ph.D. from UW CSE in theory  
Fourth year as teaching faculty

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## TAs

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Ben Zhang

Muru Zhang

Jolie Zhou

# Logistics

There are two lectures, MWF

A lecture 9:30-10:20, B lecture 1:30-2:20

Both will be recorded, with recordings posted when possible.

Try to attend your officially registered lecture, but you don't need permission to occasionally attend the other.

The lectures should be nearly identical, but there will be small differences (e.g. more/fewer/different questions)

# Logistics

Sections meet on Thursdays (starting this week)

Please go to your assigned section.

If you can't make your assigned section one week, you can ask the TA(s) in charge of another for permission to join.

Sections will **not** be recorded – we want you to be able to ask questions and give feedback without worrying about being recorded.

Handouts and solutions will be posted.

# Syllabus

When in doubt, it's on the webpage:

(or it will be soon 😊 )

<https://courses.cs.washington.edu/courses/cse312/24sp/>

# Work

## Concept Checks (7.5%)

Short "quiz" for each lecture on gradescope; identify misconceptions right away.

Due the morning of the next lecture. First due date Friday

## Section Participation (5%)

Go to section and work on problems there or (if you can't make it) do the corresponding problems on your own and email them to your TA.

## Approximately 8 Homeworks (50%)

Mostly written problems, but a few programming questions.

## Midterm (12.5%)

Evening exam Monday April 29

## Final exam (25%)

Monday June 3 12:30-2:20 Combined exam for both sections

# Communication

Ed Discussion board will be our primary means of communication.

Please check frequently.

We'll send announcement emails via Ed.

If you want to contact us:

- Private post on Ed (seen by TAs and Robbie, but not other students)

- Email Robbie

- Anonymous Feedback form on webpage



# Collaboration Policy

PLEASE collaborate! Please talk to each other and work with each other.  
(subject to the policy – details on webpage)

# What is This Class?

We're going to learn fundamentals of probability theory.

A **beautiful** and *useful* branch of mathematics.

Applications in:

Machine Learning

Natural Language Processing

Cryptography

Error-Correcting Codes

Data Structures

Data Compression

Complexity Theory

Algorithm Design

...

# Content

## Combinatorics (*fancy* counting)

Permutations, combinations, inclusion-exclusion, pigeonhole principle

## Formal definitions for Probability

Probability space, events, conditional probability, independence, expectation, variance

## Common patterns in probability

Equations and inequalities, "zoo" of common random variables, tail bounds

## Continuous Probability

pdf, cdf, sample distributions, central limit theorem, estimating probabilities

## Applications

Across CS, but with some focus on ML.

# Themes

Precise mathematical communication

Both reading and writing dense statements.

Probability in the “real world”

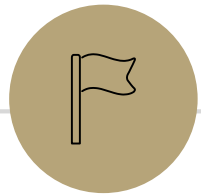
A mix of CS applications

And some actual “real life” ones.

Refine your intuition

Most people have some base level feeling of what the chances of some event are.

We’re going to train you to have better gut feelings.



# Counting



# Why Counting?

Sometimes useful for algorithm analysis.

The easiest code to write for "find X" is "try checking every spot where X could be"

"Given an array, find a set of elements that sum to 0"

"Given an array, find a set of 2 elements that sum to 0"

Gut check of "we can 'brute force' this or we can't" is super useful.

A building block toward probability theory

"What are the chances" is usually calculated by

$$= \frac{\text{how many ways can I succeed}}{\text{how many ways can I succeed} + \text{how many ways can I fail}}$$

# Remember sets?

A set is an unordered list of elements, ignoring repeats.

{1,2,3} is a set. It's the same set as {2,1,3}.

{1,1,2,3} is a very confusing way of writing the set {1,2,3}.

The cardinality of a set is the number of elements in it.

{1,2,3} has cardinality 3

$$|\{1,2,3\}| = 3.$$

# Counting Rules

$$|A \cup B| = |A| + |B|$$

← as long as  $A \cap B = \emptyset$

How many options do I have for dinner?

I could go to Chili's where there are 3 meals I choose from, or I could go to Thaiger Room where there are 5 meals I choose from (and none of them are the same between the two).

How many total choices?

$$8 = 3 + 5$$

$$3 + 5 = 8$$

Sum Rule: If you are choosing one thing between  $n$  options in one group and  $m$  in another group with no overlap, the total number of options is:  $n + m$ .



# Counting Rules

I'm still hungry...

I decide to make a sandwich. My sandwiches are always:

One of three types of bread (white, wheat, or sourdough).

One of two spreads (mayo or mustard)

One of four cheeses (American, cheddar, swiss, or Havarti)

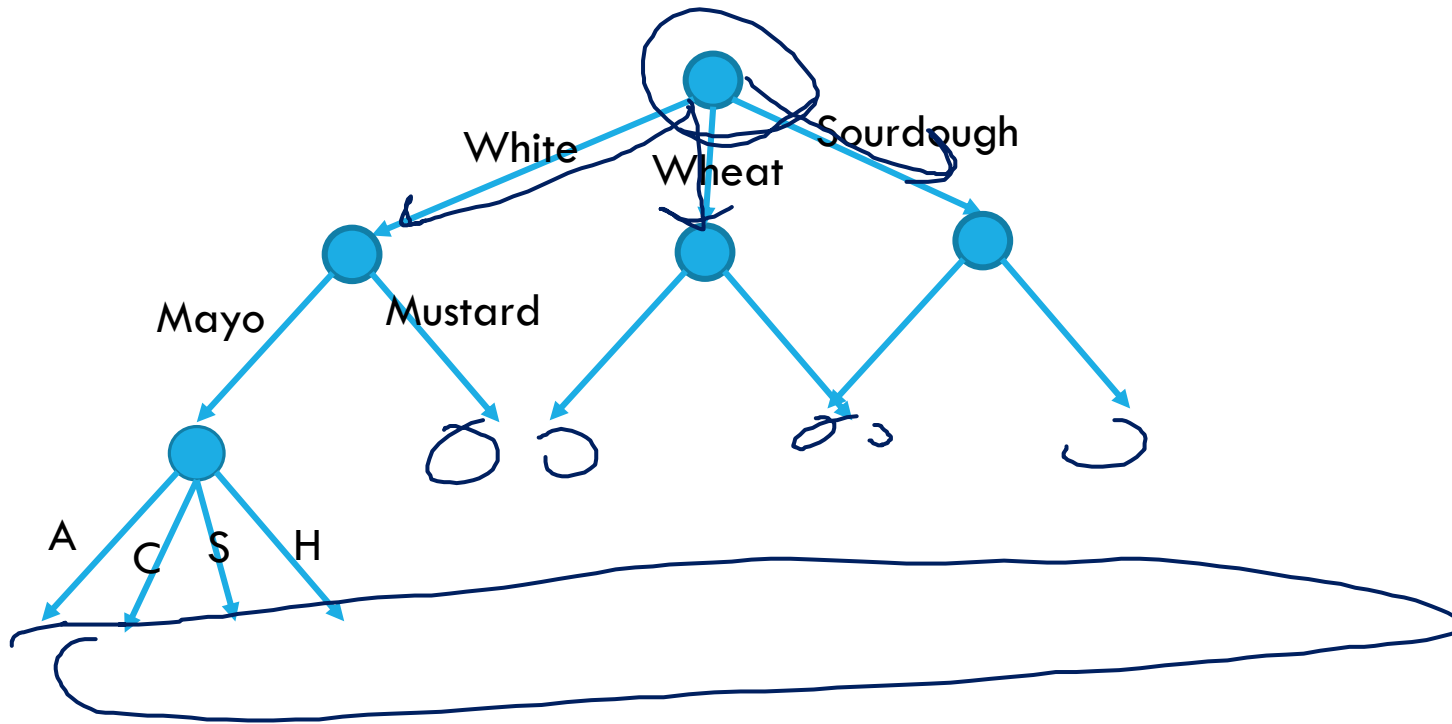
How many sandwiches can I make?

$$\begin{array}{r} 3 \\ 2 \\ 4 \end{array}$$

$$3 \times 2 \times 4$$

$$24$$

# Sandwiches



Step 1: choose one of the three breads.

Step 2: regardless of step 1, choose one of the two spreads.

Step 3: regardless of steps 1 and 2, choose one of the four cheeses.

$$3 \cdot 2 \cdot 4 = 24.$$

# Counting Rules

Sum Rule: If you are choosing one thing between  $n$  options in one group and  $m$  in another group with no overlap, the total number of options is:  $n + m$ .

Product Rule: If you have a sequential process, where step 1 has  $n_1$  options, step 2 has  $n_2$  options, ..., step  $k$  has  $n_k$  options, and you choose one from each step, the total number of possibilities is  $n_1 \cdot n_2 \cdots n_k$

# Applications of the product rule

Remember Cartesian products?

$$S \times T = \{(x, y) : x \in S, y \in T\}$$

$$\{1, 2\} \times \{a, b, c\} = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

$B \times S \times C$

How big is  $S \times T$ ? (i.e. what is  $|S \times T|$ ?)

Step 1: choose the element from  $S$ .

Step 2: choose the element from  $T$ .

Total options:  $|S| \cdot |T|$

# Power Sets

$$\mathcal{P}(S) = \{X: X \subseteq S\}$$

$$\mathcal{P}(\{1,2,3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

How many subsets are there of  $S$ , i.e. what is  $|\mathcal{P}(S)|$ ?

# Power Sets

$$\mathcal{P}(S) = \{X: X \subseteq S\}$$

$$\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Handwritten calculation for the number of subsets of  $\{1,2,3\}$ :

$$3 + 1 + 3 + 3 + 1 = 8$$

How many subsets are there of  $S$ , i.e. what is  $|\mathcal{P}(S)|$ ?

$$\text{If } S = \{e_1, e_2, \dots, e_{|S|}\}$$

Step 1: is  $e_1$  in the subset? 2

Step 2: is  $e_2$  in the subset? 2

...

Step  $|S|$ : is  $e_{|S|}$  in the subset? 2

$2 \cdot 2 \cdots 2$ ,  $|S|$  times, i.e.,  $2^{|S|}$ .

Handwritten calculation:  $2^3 = 8$

# Baseball Outfits

The Husky baseball team has three hats (purple, black, gray)

Three jerseys (pinstripe, purple, gold)

And three pairs of pants (gray, white, black)

How many outfits are there (consisting of one hat, jersey, and pair of pants) if

the pinstripe jersey cannot be worn with gray pants,  
the purple jersey cannot be worn with white pants,  
and the gold jersey cannot be worn with black pants.

# Baseball Outfits

Step 1: 3 choices for hats,

Step 2: 3 choices for jerseys

Step 3:...



# Baseball Outfits

Step 1: 3 choices for hats.

Step 2: 3 choices for jerseys.

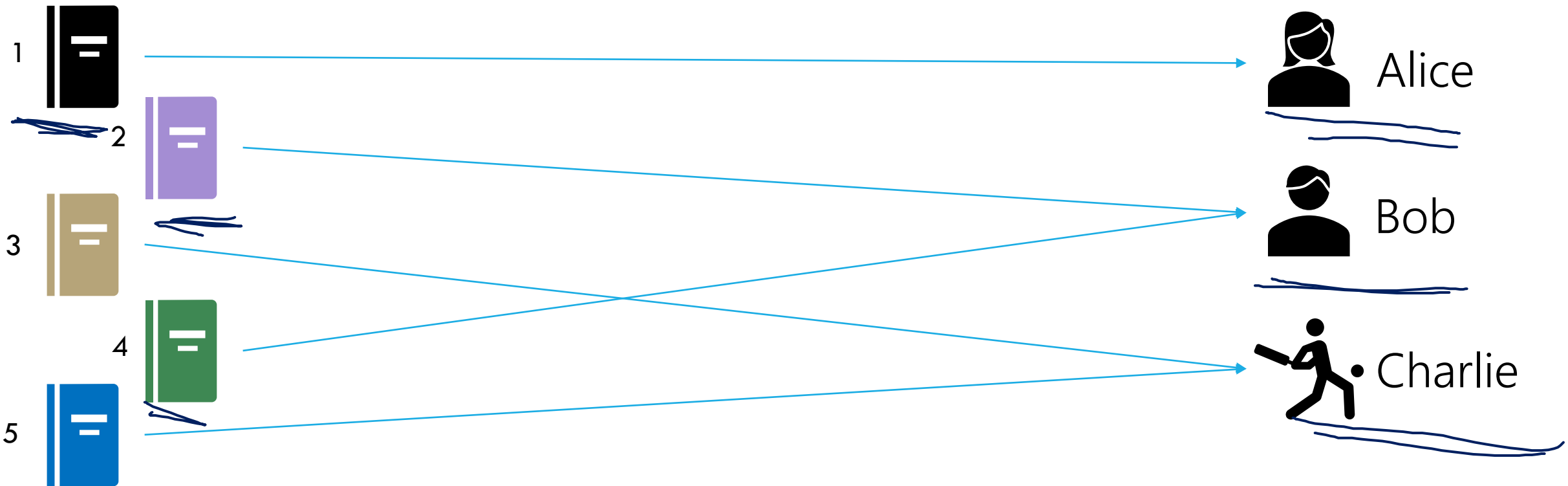
Step 3: Regardless of which jersey we choose, we have 2 options for pants (even though there are three options overall).

$$\underset{\downarrow}{3} \cdot \underset{\downarrow}{3} \cdot \underset{\downarrow}{2} = \underset{\downarrow}{18}.$$

# Assigning Books

We have 5 books to split to 3 people (Alice, Bob, and Charlie)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).



# Assigning Books

We have 5 books to split to 3 people (Alice, Bob, and Charlie)

Every book goes to exactly one person, but each person could end up with no books (or all of them, or something in between).

Attempt 1: We're choosing subsets!

Alice could get any of the  $2^5 = 32$  subsets of the books.

Bob could get any of the  $2^5 = 32$  subsets of the books.

Charlie could get any of the  $2^5 = 32$  subsets of the books.

Total is product of those three steps  $32 \cdot 32 \cdot 32 = 32768$

# Activity

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Fill out the poll everywhere so Robbie  
can adjust his explanation

Go to [pollev.com/robbie](https://pollev.com/robbie) and login  
with your UW identity

# Activity

Attempt 1: We're choosing subsets!

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We overcounted!

If Alice gets  $\{1,2\}$ , Bob can't get any subset, he can only get a subset of  $\{3,4,5\}$ . And Charlie's subset is just whatever is leftover after Alice and Bob get theirs...

# Fixing All The Books

You could

List out all the options for Alice.

For each of those (separately), list all the possible options for Bob and Charlie.

Use the Summation rule to combine.

~OR~ you could come at the problem from a different angle.

# Fixing All the Books

Instead of figuring out which books Alice gets, choose book by book which person they go to.

Step 1: Book 1 has 3 options (Alice, Bob, or Charlie).

Step 2: Book 2 has 3 options (A, B, or C)

...

Step 5: Book 5 has 3 options.

Total:  $3^5$ .

# More sequence practice

How many length 3 sequences are there consisting of distinct elements of  $\{1,2,3\}$ .



# Pause

Questions in combinatorics and probability are often dense. A single word can totally change the answer. Does order matter or not? Are repeats allowed or not? What makes two things “count the same” or “count as different”?

Let’s look for some keywords

How many length 3 sequences are there consisting of distinct elements of  $\{1,2,3\}$ .

**Sequences** implies that order matters –  $(1,2,3)$  and  $(2,1,3)$  are different.

**Distinct** implies that you can’t repeat elements  $(1,2,1)$  doesn’t count.

$\{1,2,3\}$  is our “universe” – our set of allowed elements.

# More sequence practice

How many length 3 sequences are there consisting of distinct elements of  $\{1,2,3\}$ .

Step 1: 3 options for the first element.

Step 2: 2 (remaining) options for the second element.

Step 3: 1 (remaining) option for the third element.

$$3 \cdot 2 \cdot 1.$$

# Factorial

That formula shows up a lot.

The number of ways to “permute” (i.e. “reorder”, i.e. “list without repeats”)  $n$  elements is “ $n$  factorial”

**$n$  factorial**

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 1$$

We only define  $n!$  for natural numbers  $n$ .

As a convention, we define:  $0! = 1$ .



**More Practice**

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# Strings

How many strings of length 5 are there over the alphabet  $\{A, B, C, \dots, Z\}$ ? (repeated characters allowed)

How many binary strings of length  $n$  are there?

# Strings

How many strings of length 5 are there over the alphabet  $\{A, B, C, \dots, Z\}$ ? (repeated characters allowed)

$$26^5$$

How many binary strings of length  $n$  are there?

$$2^n$$