CSE 312

Foundations of Computing II

Lecture 1: Introduction & Counting

https://cs.washington.edu/312

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See https://cs.washington.edu/312/staff.html to learn more about their backgrounds and interests!

Lectures and Sections

- **Lectures MWF (ARC 147)**
  - 1:30-2:20pm
  - Lecture recording/streaming will be available!
  - Slides are also available.
- **Poll Everywhere**
  - We will sometimes use Poll Everywhere during class
  - You sign up directly
- **Sections Thu (starts this week)**
  - Not recorded
  - Will prepare you for problem sets!

Go to https://www.polleverywhere.com/login and login using YOURNETID@uw.edu

Questions and Discussions

- **Office hours throughout the week (starting this Friday)**
  - See https://cs.washington.edu/312/staff.html
- **Ed Discussion**
  - You should have received an invitation (synchronized with the class roster)
  - Material (resources tab)
  - Announcements (discussion tab)
  - Discussion (discussion tab)

Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructor for personal issues.

Engagement

- **“Concept checks” after each lecture 5-8 %**
  - Must be done (on Gradescope) before the next lecture by 1:00 pm.
  - Simple questions to reinforce concepts taught in each class
  - Keep you engaged throughout the week, so that homework becomes less of a hurdle
- **9 Problem Sets (Gradescope) 45-50 %**
  - Solved individually. Discussion with others allowed but separate solutions
  - Generally due Wednesdays starting next week, except for midterm week but Fridays after Thanksgiving
  - First problem set posted later today
- **Midterm 15-20 %**
  - In class on **Wednesday, October 30**
- **Final Exam 30-35 %**
  - **Monday, December 9** at **2:30-4:20 pm** in this room (as in UW Autumn Quarter Exam Schedule)

Check out the syllabus for policies on late submission for checkpoints and HW

Foundations of Computing II

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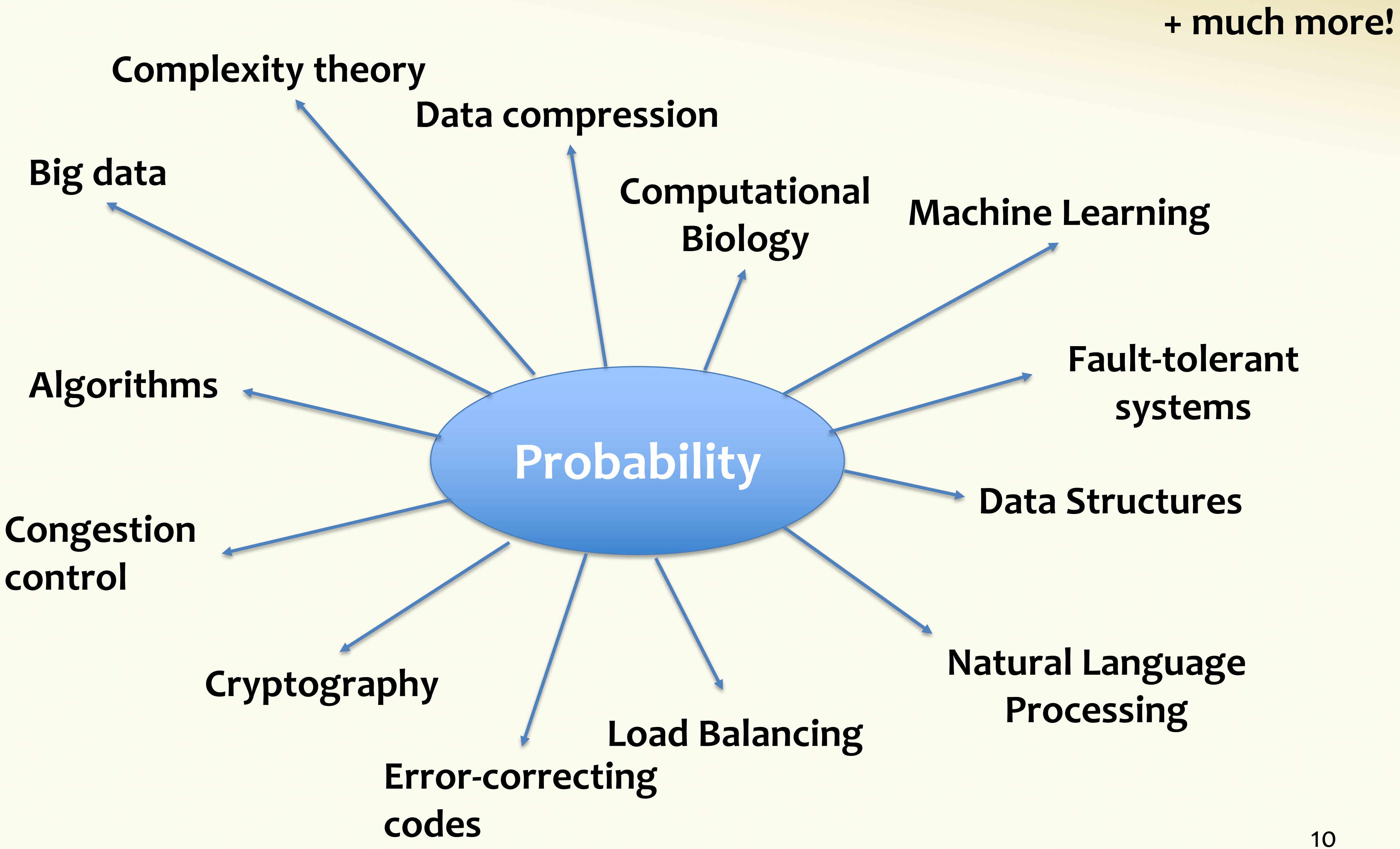
Intro to Counting, Probability & Statistics

Probability

- The math to understand randomness
- Best way to understand complex systems even if there is no randomness!

Probability in Computer Science

- Understanding random inputs to algorithms
  - ML, program testing, algorithm analysis, …
- Understanding hidden information
  - Cryptography, privacy, fault tolerance, computer security, …
- Efficient algorithms and systems design
  - Data structures, systems, algorithms, ML, …
- …



+ much more!
Complexity theory
Data compression
Big data
Computational Biology
Machine Learning
Algorithms
Fault-tolerant systems
Probability
Data Structures
Congestion control
Cryptography
Natural Language Processing
Load Balancing
Error-correcting codes

Content

- Counting (basis of discrete probability)
  - Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle
- What is probability
  - Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance
- Properties of probability
  - Various inequalities, Zoo of discrete random variables, Concentration, Tail bounds
- Continuous Probability
  - Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation
- Applications
  - A sample of randomized algorithms, differential privacy, learning …

Today: A fast introduction to counting so you will have enough to work on in section tomorrow…

How to count objects with a property?

"How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?"

Abstractly: What is the size of a set S?

(Discrete) Probability and Counting are the same

"Probability that a uniformly random student has black hair?"

$$= \frac{\#\ students\ with\ black\ hair}{\#students}$$

Today – Two basic rules

- Sum rule
- Product rule

Sum Rule

If you can choose from

- EITHER one of n options,
- OR one of m options with NO overlap with the previous n

then the number of possible outcomes of the experiment is

$$n + m$$

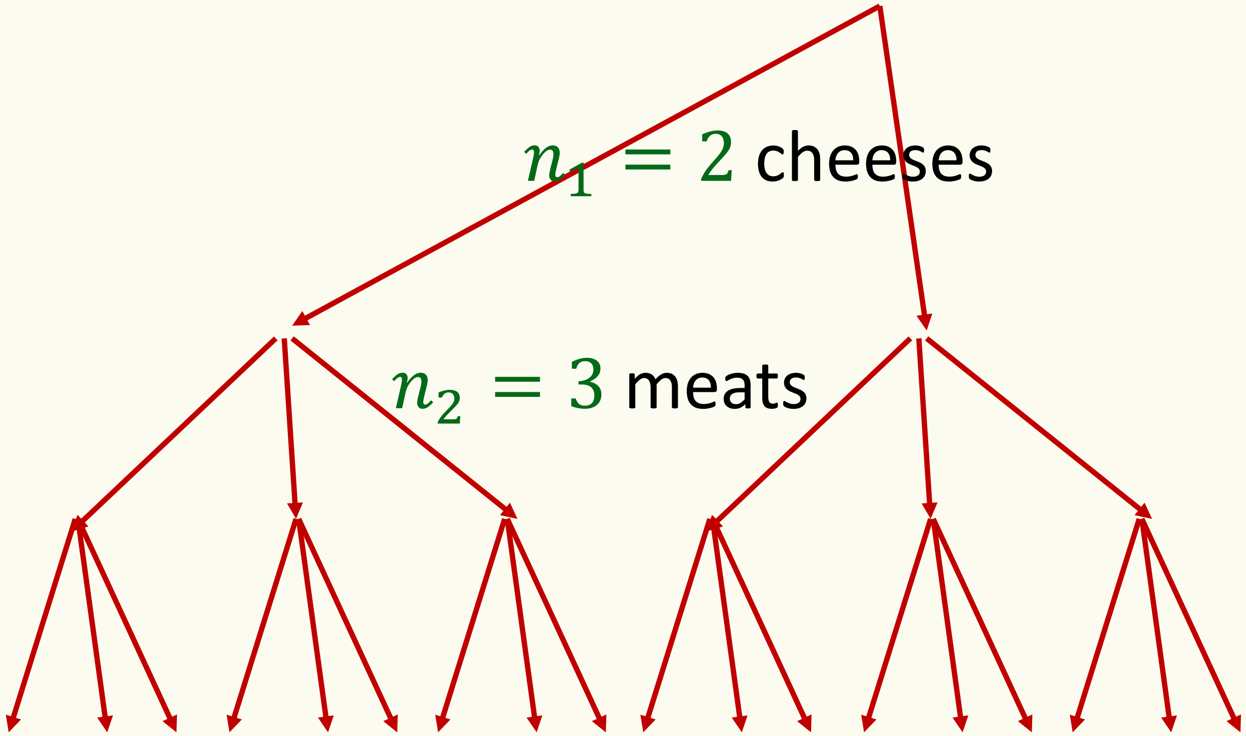
Counting "lunches"

If a lunch order consists of **either** one of 6 soups **or** one of 9 salads, how many different lunch orders are possible?

Product Rule: In a sequential process, there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), …, and
- n_m choices for the m^{th} step (given the previous choices),

then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$



$n_3 = 3$ veggies

Example: "How many subs?"

$\square \times \square \times \square = \square$

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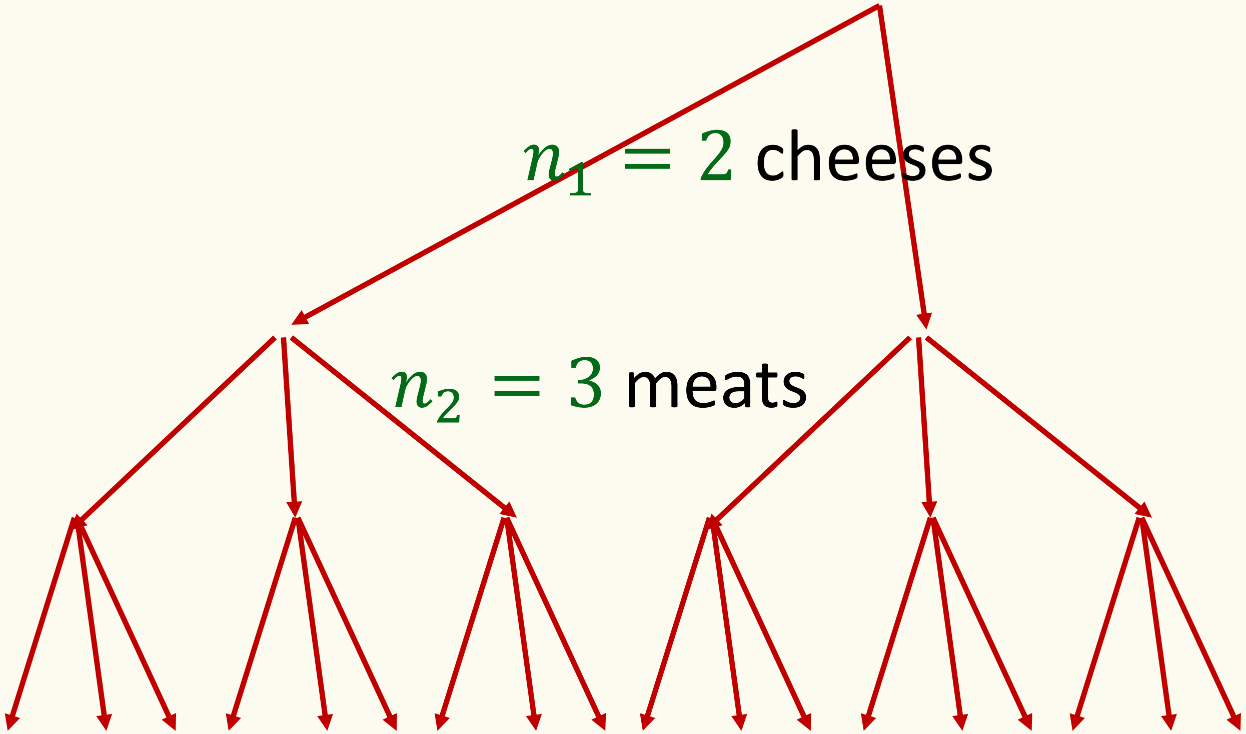

$n_3 = 3$ veggies

Example: "How many subs?"

$$\boxed{2} \times \boxed{3} \times \boxed{3} = \boxed{18}$$

Product rule examples – Strings

How many strings of length 5 over the alphabet $\{A, B, C, \dots, Z\}$ are there?

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

$\square \times \square \times \square \times \square \times \square = \square$

Product rule examples – Strings

How many strings of length 5 over the alphabet $\{A, B, C, \ldots, Z\}$ are there?

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

$$\boxed{26} \times \boxed{26} \times \boxed{26} \times \boxed{26} \times \boxed{26} = \boxed{26^5}$$

How many binary strings of length n over the alphabet $\{0,1\}$?

- E.g., $0 \cdots 0, 1 \cdots 1, 0 \cdots 01$, …

$$\boxed{} \times \boxed{} \times \boxed{} \times \cdots \times \boxed{} = \boxed{}$$

Product rule examples – Strings

How many strings of length 5 over the alphabet $\{A, B, C, \dots, Z\}$ are there?

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

$$26 \times 26 \times 26 \times 26 \times 26 = 26^5$$

How many binary strings of length n over the alphabet $\{0,1\}$?

- E.g., $0 \cdots 0$, $1 \cdots 1$, $0 \cdots 01$, …

$$2 \times 2 \times 2 \times \cdots \times 2 = 2^n$$

Product rule example – Cartesian Product

Definition. The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

$$|S \times T| = \square \times \square$$

$$|A_1 \times A_2 \times \cdots \times A_n| = \square \times \square \times \square \times \cdots \times \square$$

Product rule example – Cartesian Product

Definition. The **cartesian product** of two sets S, T is

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$$|S \times T| \;=\; \boxed{|S|} \times \boxed{|T|}$$

$$|A_1 \times A_2 \times \cdots \times A_n| \;=\; \boxed{} \times \boxed{} \times \boxed{} \times \cdots \times \boxed{}$$

Product rule example – Cartesian Product

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$$|S \times T| \quad = \quad \boxed{|S|} \times \boxed{|T|}$$

$$|A_1 \times A_2 \times \cdots \times A_n| \quad = \boxed{|\boldsymbol{A_1}|} \times \boxed{|\boldsymbol{A_2}|} \times \boxed{|\boldsymbol{A_3}|} \times \cdots \times \boxed{|\boldsymbol{A_n}|}$$

Product rule example – Power set

Definition. The **power set** of S is the set of all subsets of S,

$$\{X : X \subseteq S\}.$$

Notations: $\mathcal{P}(S)$ or simply 2^S (which we will use).

Example. $2^{\{\bigstar,\spadesuit\}} = \{\emptyset, \{\bigstar\}, \{\spadesuit\}, \{\bigstar, \spadesuit\}\}$

$2^{\emptyset} = \{\emptyset\}$

…

How many different subsets of S are there if $|S| = n$?

Product rule example – Power set

set $S = \{e_1, e_2, e_3, \cdots, e_n\}$

subset $X = \{ \qquad\qquad\qquad \}$

$$\square \times \square \times \square \times \cdots \times \square = \square$$

Product rule example – Power set

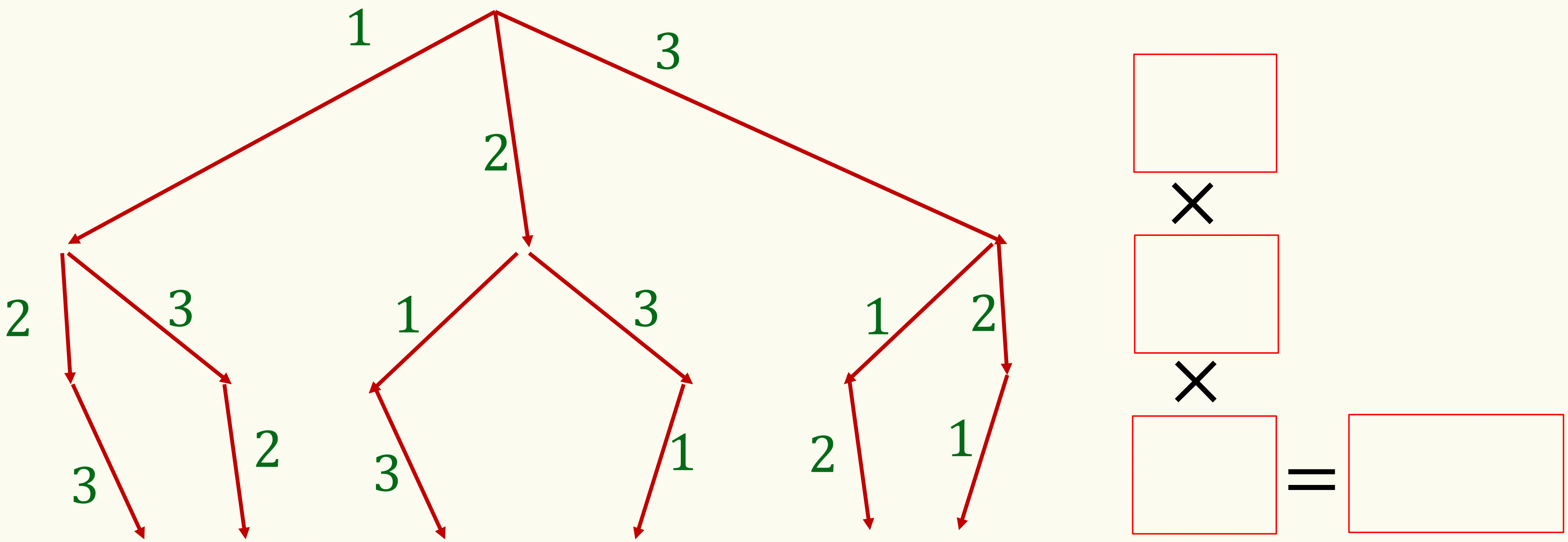
set $S = \{e_1, e_2, e_3, \cdots, e_n\}$

subset $X = \{ \qquad \}$

$$\boxed{\mathbf{2}} \times \boxed{\mathbf{2}} \times \boxed{\mathbf{2}} \times \cdots \times \boxed{\mathbf{2}} = \boxed{\mathbf{2^n}}$$

Proposition. $|2^S| = 2^{|S|}$

How many strings in $\{1,2,3\}^3$ with no repeating elements?
123, 132, 213,…



How many strings in $\{1,2,3\}^3$ with no repeating elements?

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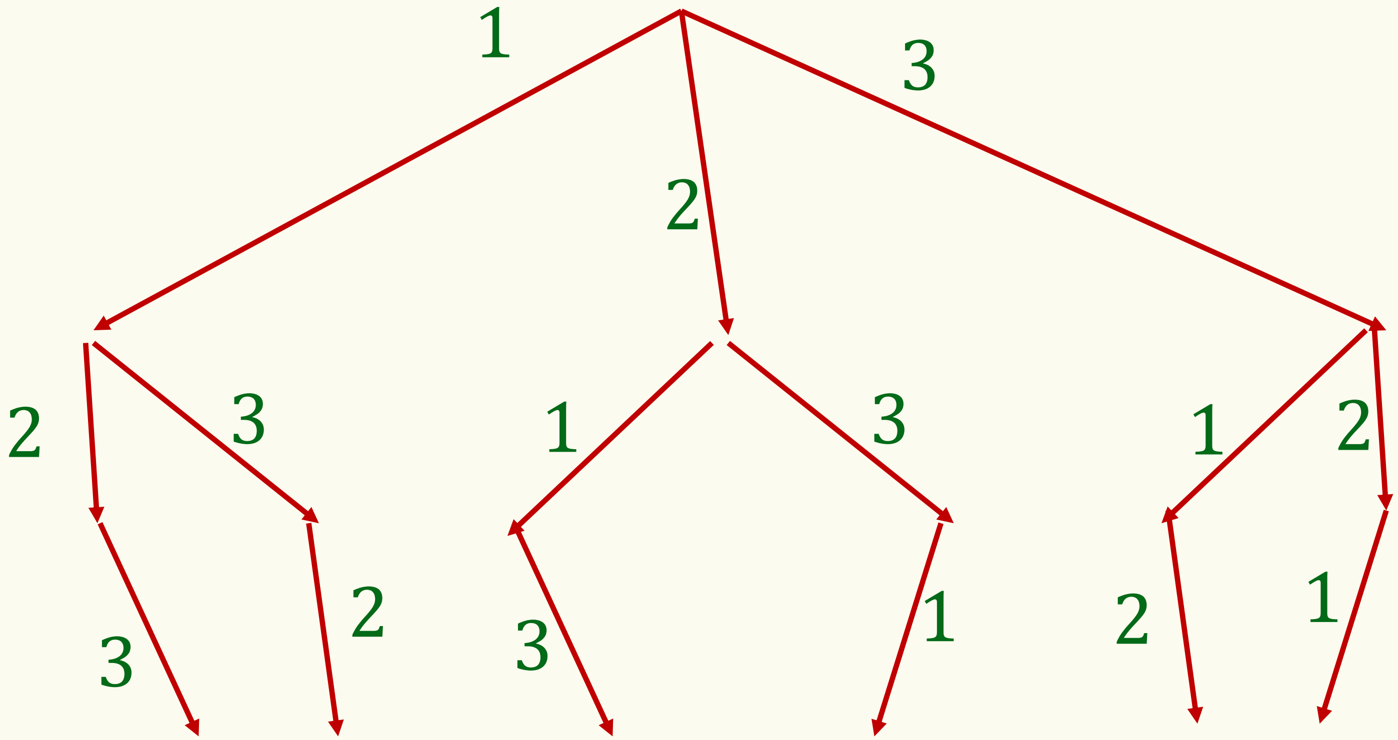


$$3 \times 2 \times 1 = 6$$

Factorial

"How many ways to order n elements?"

Permutations

Answer = $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Definition. The **factorial function** is

$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

Note: $0! = 1$

Nice use of sum rule: Counting using complements

"How many sequences in $\{1,2,3\}^3$ *have repeating elements?"* m

"# of sequences in $\{1,2,3\}^3$ *with no repeating elements"* $n = $ **3!**

"# of sequences in $\{1,2,3\}^3$ $3^3 = 27$ $= m + n$ *by the sum rule*

$m = 27 - n =$ **27-3!**

Distinct Letters

"How many sequences of 5 ***distinct*** *alphabet letters from* $\{A, B, \ldots, Z\}$*?"*

E.g., AZURE, BINGO, TANGO. But not: STEVE, SARAH

Answer: $26 \times 25 \times 24 \times 23 \times 22 = 7893600$

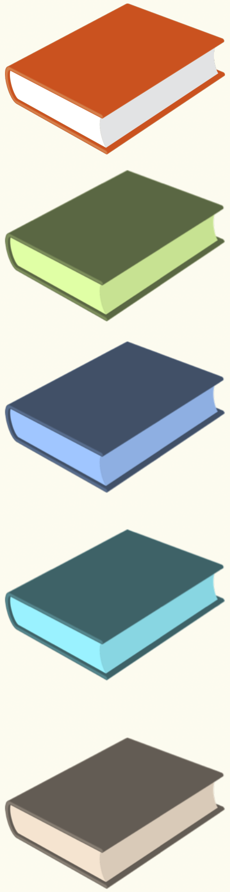
In general

Fact. # of k-element sequences of distinct symbols from an n-element set is

$$P(n,k) = n\times(n-1)\times\cdots\times(n-k+1) = \frac{n!}{(n-k)!}$$

Product rule – One more example

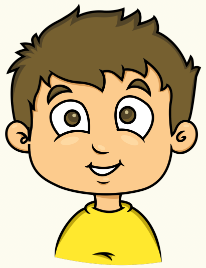
5 books



> ***"How many ways are there to distribute 5 books among Alice, Bob, and Charlie?"***

Every book to one person, everyone gets ≥ 0 books.

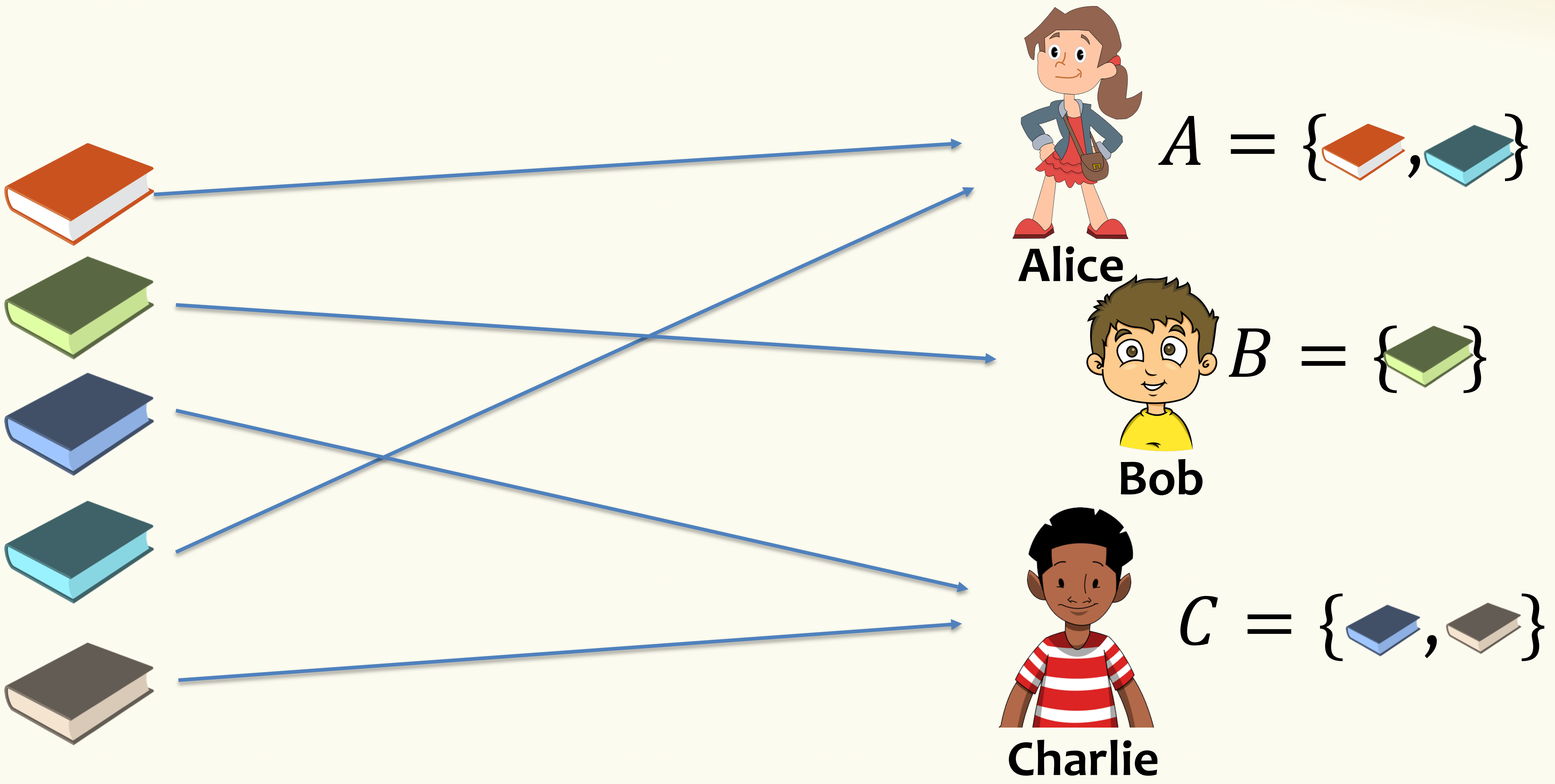
Alice



Bob

Charlie

Example Book Assignment



Book assignment – Modeling

2^5 options

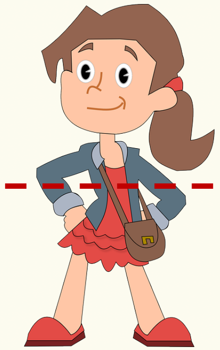
$\times$

2^5 options

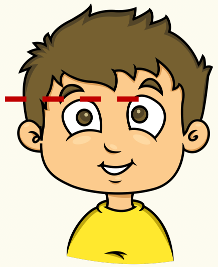
$\times$

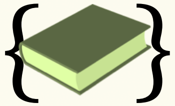
2^5 options

$= 2^{15}$ assignments



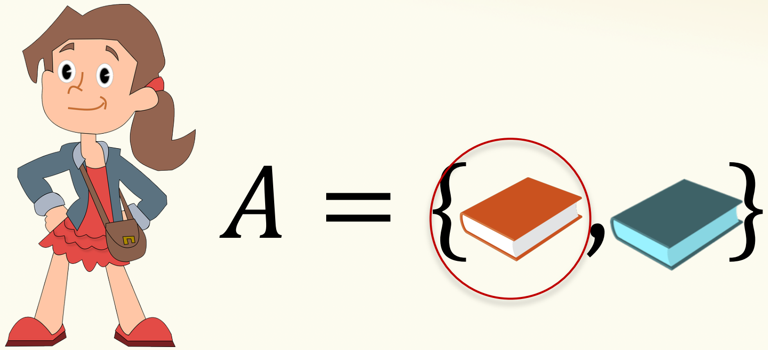
$A = \{$  $\}$



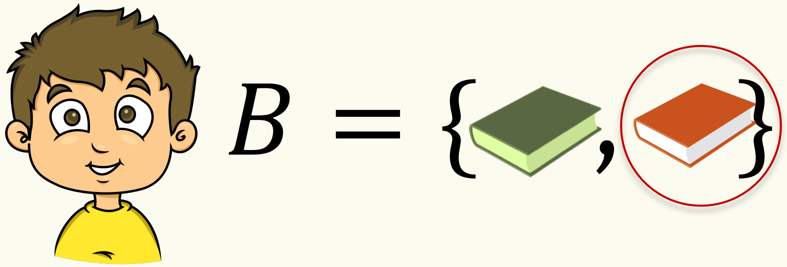
$B = \{$  $\}$

$C = \{$  $\}$

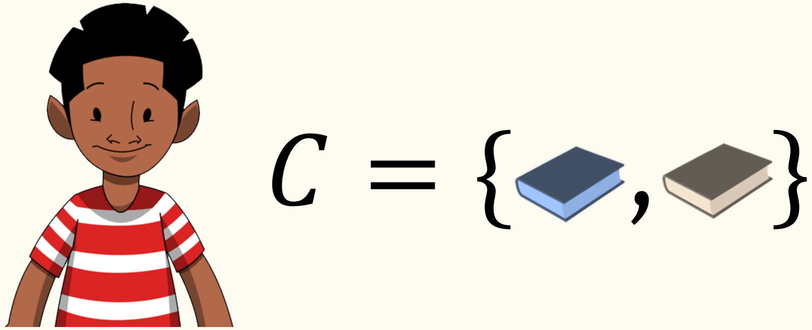
Problem – Overcounting



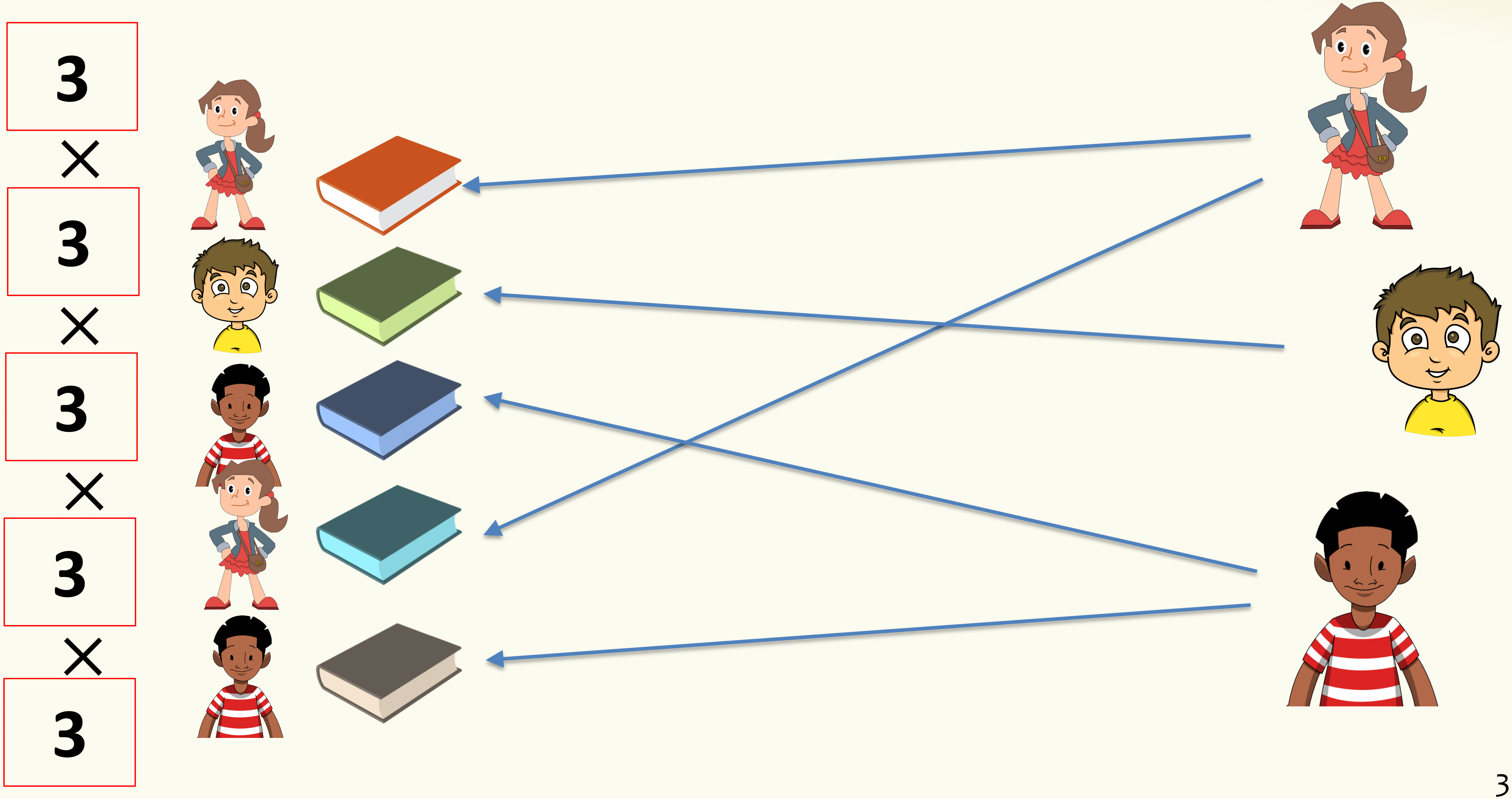
Problem: We are counting some <u>invalid</u> assignments!!!
$\longrightarrow$ **<u>overcounting</u>!**



What went wrong in the sequential process?
After assigning A to Alice,
B is no longer a valid option for Bob



Book assignments – A Clever Approach



Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself!
Think about counter examples to your own solution!
Start with a smaller version of the same problem!

The first concept check is out and due at 1:00pm before Friday’s lecture