

PROBABILITY

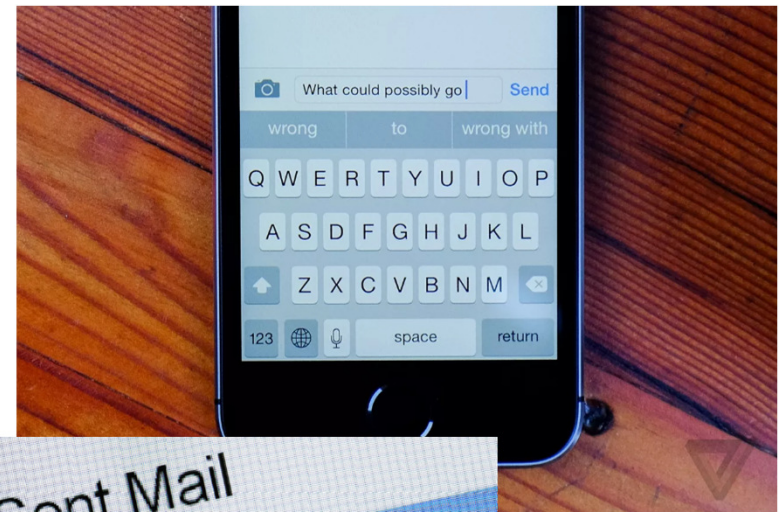
THE NAIVE BAYES CLASSIFIER

LUXI WANG, PEMI NGUYEN,
MITCHELL ESTBERG AND SHREYA JAYARAMAN
ALEX TSUN




AGENDA

- WHAT IS MACHINE LEARNING?
- FEATURIZING EMAILS
- NAIVE BAYES

MACHINE LEARNING IN THE REAL WORLD



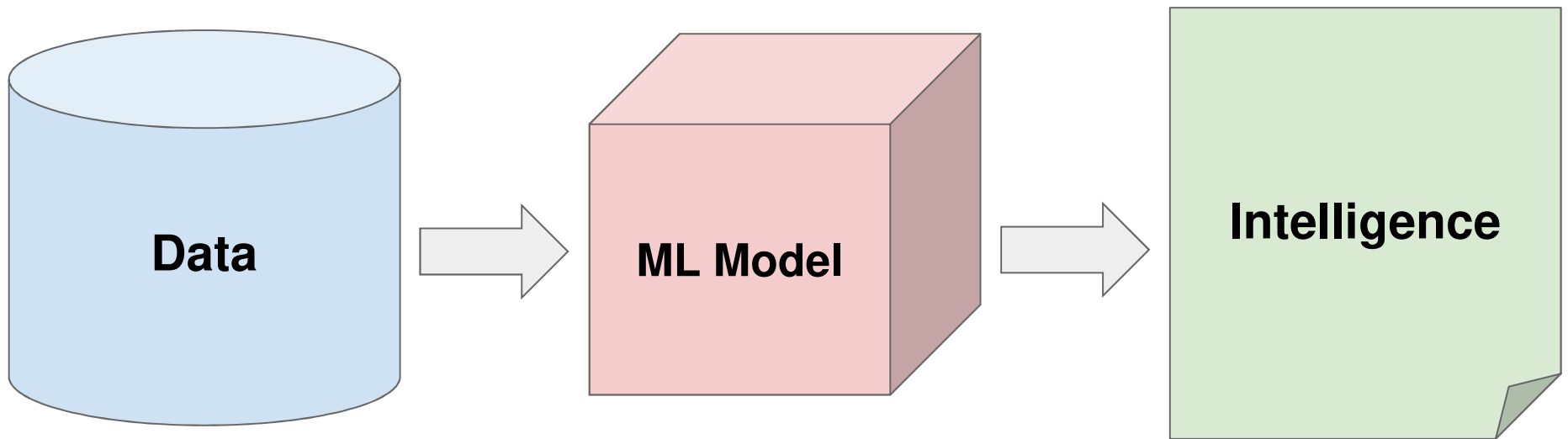
Jobs you may be interested in

 Engineering Manager - Data Infrastructure Twilio Inc. — San Francisco, CA, US View Job	 Chief Architect Appthority — San Francisco Bay Area View Job	 Sr. Engineering Manager Comcast Silicon Valley Innovation Center — San View Job
--	--	---

[See more jobs >](#)








ML PIPELINE








From **Wikipedia**: “Machine learning is the study of computer algorithms that improve automatically through experience.”

YOU ARE A MACHINE!

Number	Shape	“Label”
3		12
5		15
-2		-8
7		21
-4		???

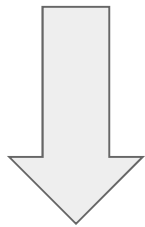
Given examples with correct “labels”, make predictions!

YOU ARE A MACHINE!

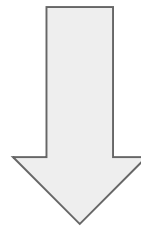
Number	Shape	“Label”
3		12
5		15
-2		-8
7		21
-4		-16

Given examples with correct “labels”, make predictions!

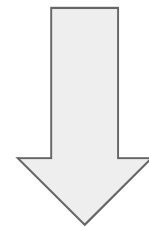
REGRESSION: IDEA



\$ 340,135

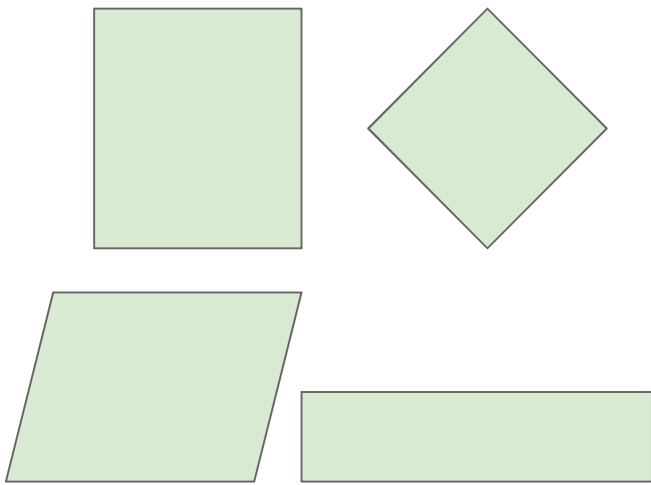


\$801,353

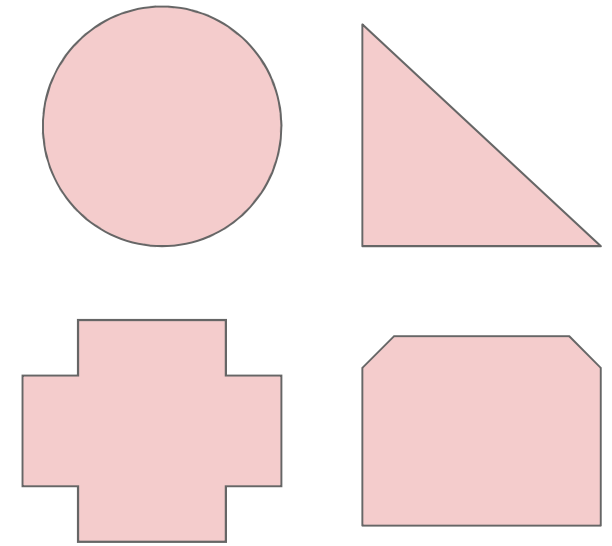


??????

CLASSIFICATION: IDEA

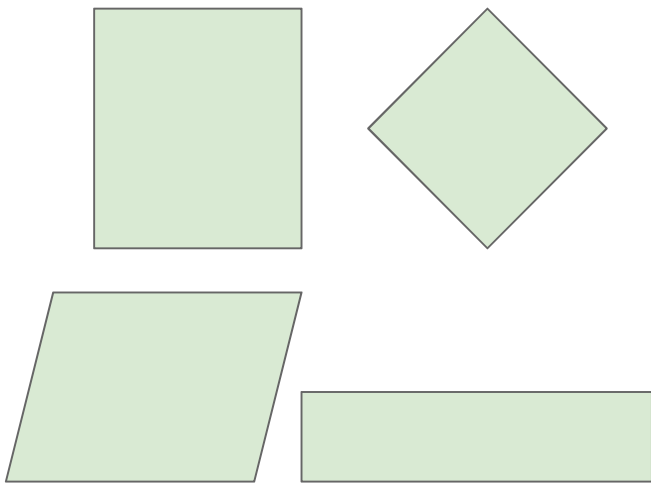


“Green” class

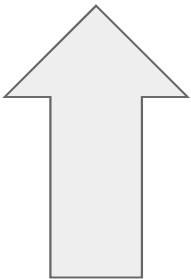


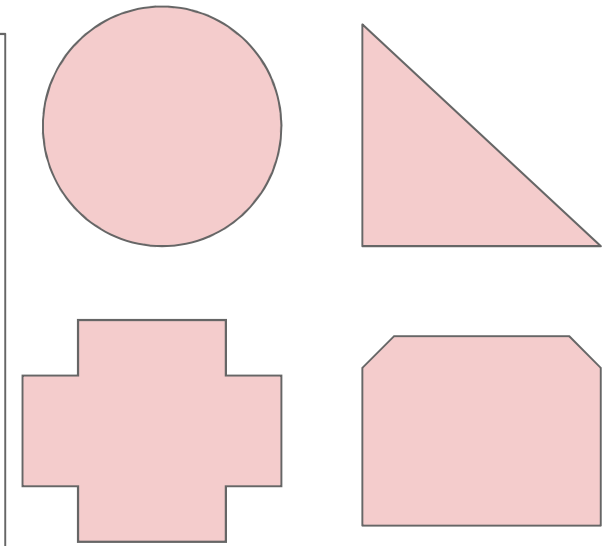
“Red” class

CLASSIFICATION: IDEA



“Green” class


Is this new shape
supposed to be
“green” or “red”?

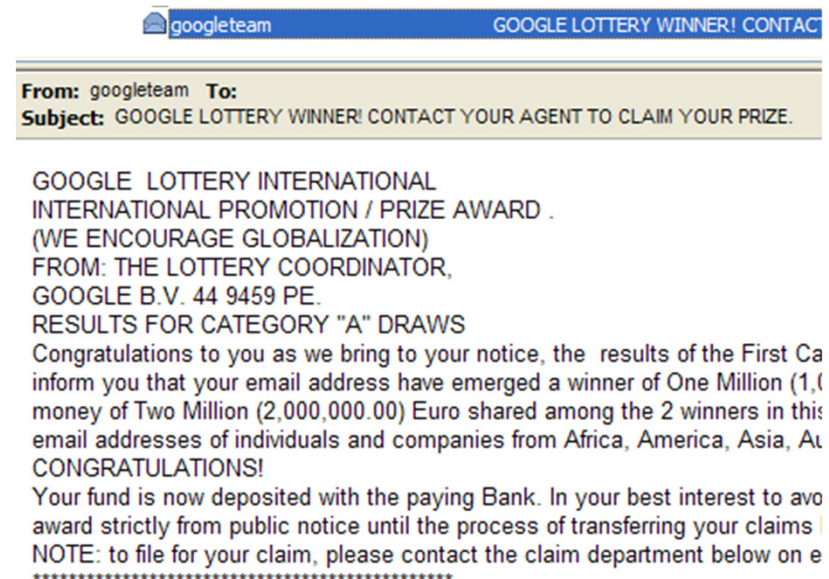
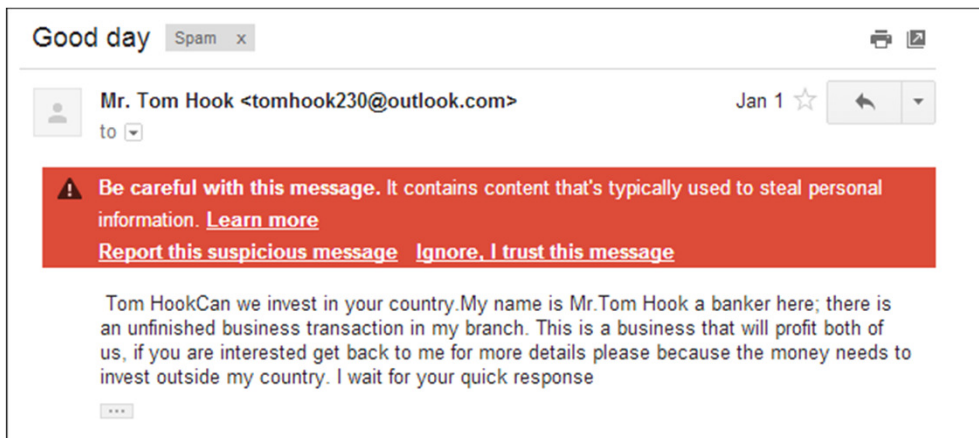


“Red” class



SPAM FILTER

- In real life, you may have seen a lot of spam emails like this.
- Building a good spam filter helps protect users from potential scams, unnecessary advertising, or malware links.



EVALUATING PERFORMANCE

Training Set

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

Test Set

Email	Label
You buy valium!	Spam
You need valium sir.	Spam
I hope you are healthy.	Ham
...	...
...	...

We “**train**” our spam filter on the training set, and **evaluate** performance using a test set (data that is unseen by the spam filter initially). This gives an unbiased estimate of performance.

SPAM FILTER TASK

Training Set

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham



Predict whether this email is spam or ham:

You buy Valium!

EMAILS AS WORD COLLECTIONS

Email	Set of Words in the Email
<p>SUBJECT: Top Secret Business Venture</p> <p>Dear Sir. First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret...</p>	<p>{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virtue, of, its, nature, as, being, utterly, confidential, and}</p>

For simplicity, we will

- Ignore Duplicate Words
- Ignore Punctuation
- Ignore Casing

EMAILS AS WORD COLLECTIONS

Email	Set of Words in the Email
SUBJECT: Top Secret Business Venture Dear Sir. First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret...	{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virtue, of, its, nature, as, being, utterly, confidential, and}
Hello hello hello there.	{hello, there}

For simplicity, we will

- Ignore Duplicate Words
- Ignore Punctuation
- Ignore Casing

EMAILS AS WORD COLLECTIONS

Email	Set of Words in the Email
SUBJECT: Top Secret Business Venture Dear Sir. First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret...	{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virtue, of, its, nature, as, being, utterly, confidential, and}
Hello hello hello there.	{hello, there}
You buy Valium!	{you, buy, valium}

For simplicity, we will

- Ignore Duplicate Words
- Ignore Punctuation
- Ignore Casing

OUR APPROACH

Compute and Compare:

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$\mathbb{P}(\text{ham} \mid \text{"You buy Valium!"})$$

Then predict whichever is larger! Can we get away with just computing one of them?

OUR APPROACH

Compute and Compare:

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

$$\mathbb{P}(\text{ham} \mid \text{"You buy Valium!"})$$

Then predict whichever is larger! Can we get away with just computing one of them?

Equivalently, note that these add to 1, so we can just compute

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$$

and if it is greater than 0.5, then we predict **spam**.

Otherwise, we predict **ham**.

Note: We resolve the tie in favor of **ham**.

NAIVE BAYES CLASSIFIER - THE BAYES PART

Bayes Theorem:
$$\mathbb{P}(A | B) = \frac{\mathbb{P}(B | A) \mathbb{P}(A)}{\mathbb{P}(B)}$$



Apply it to our example:

$$\mathbb{P}(\text{spam} | \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} | \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

NAIVE BAYES CLASSIFIER - WHAT WE CALCULATE

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

NAIVE BAYES CLASSIFIER - WHAT WE CALCULATE

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \quad [\text{LTP}]$$

NAIVE BAYES CLASSIFIER - WHAT WE CALCULATE

$$\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"}) = \frac{\mathbb{P}(\text{"You buy Valium!"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Valium!"})}$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \quad [\text{LTP}]$$

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}}$$

$$\mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

(our approximation for these probabilities,
based on the training set)

NAIVE BAYES CLASSIFIER - THE NAIVE PART

It is somewhat unlikely that we have the email "You buy Valium!" in our training data. (In this case we don't!)

NAIVE BAYES CLASSIFIER - THE NAIVE PART

It is somewhat unlikely that we have the email "You buy Valium!" in our training data. (In this case we don't!)

We naively assume that words are conditionally independent from each other, given the label (In reality, they aren't):

NAIVE BAYES CLASSIFIER - THE NAIVE PART

It is somewhat unlikely that we have the email "You buy Valium!" in our training data. (In this case we don't!)

We naively assume that words are conditionally independent from each other, given the label (In reality, they aren't):

$$\mathbb{P}(\{"you", "buy", "valium"\} | \text{spam})$$

$$\approx \mathbb{P}("you" | \text{spam})\mathbb{P}("buy" | \text{spam})\mathbb{P}("valium" | \text{spam})$$

NAIVE BAYES CLASSIFIER - THE NAIVE PART

It is somewhat unlikely that we have the email "You buy Valium!" in our training data. (In this case we don't!)

We **naively assume that words are conditionally independent from each other, given the label (In reality, they aren't):**

$$\begin{aligned} & \mathbb{P}(\{"you", "buy", "valium"\} | \text{spam}) \\ & \approx \mathbb{P}("you" | \text{spam}) \mathbb{P}("buy" | \text{spam}) \mathbb{P}("valium" | \text{spam}) \end{aligned}$$

Then we estimate for example that

$$\mathbb{P}("you" | \text{spam}) = \frac{\text{number of spam emails containing "you" (in training set)}}{\text{number of spam emails (in training set)}}$$

WHY IS THIS NAIVE?

Consider for example the following two emails:

“!!!Lunch free for You!!!!!!” *Spam*

“You free for lunch?” *Ham*

WHY IS THIS NAIVE?

Consider for example the following two emails:

“!!!Lunch free for You!!!!!” *Spam*

“You free for lunch?” *Ham*

One shortfall of our model is that it will make the same prediction for these since they have the same set of words!

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) =$$

$$\mathbb{P}(\text{ham}) =$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"you"} \mid \text{ham}) =$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"buy"} \mid \text{ham}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) =$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) =$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$\begin{aligned}
 &= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 &= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}
 \end{aligned}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = 0$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = 1 \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1}{2}$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = 0$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = 1 \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1}{2}$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = 0$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = 1 \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1}{2}$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$\begin{aligned}
 &= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 &= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 &= \mathbf{1}
 \end{aligned}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = 0$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = 1 \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1}{2}$$



EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

= 1 (Marked as spam since no ham email contained "buy")

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1}{3} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = 0$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = 1 \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1}{2}$$



WHAT HAPPENS IF WE GOT A 0?

$P(\text{"You buy Valium!"} \mid \text{ham}) = 0$ since $P(\text{"buy"} \mid \text{ham}) = 0$, since no ham email in our training data contained the word **'buy'**.

But does that mean we will never encounter a ham email with word **'buy'**?



What about the ham:
"I'll buy sunflowers"

LAPLACE SMOOTHING

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i



LAPLACE SMOOTHING

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{|\text{total spam emails (training set) containing } w_i| + 1}{|\text{total spam emails (training set)}| + 2}$$



LAPLACE SMOOTHING

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Same for ham emails.

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{|\text{total spam emails (training set) containing } w_i| + 1}{|\text{total spam emails (training set)}| + 2}$$

$$\mathbb{P}(w_i \mid \text{ham}) = \frac{|\text{total ham emails (training set) containing } w_i| + 1}{|\text{total ham emails (training set)}| + 2}$$



LAPLACE SMOOTHING



Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Same for ham emails.

$$\mathbb{P}(w_i \mid \text{spam}) = \frac{|\text{total spam emails (training set) containing } w_i| + 1}{|\text{total spam emails (training set)}| + 2}$$

$$\mathbb{P}(w_i \mid \text{ham}) = \frac{|\text{total ham emails (training set) containing } w_i| + 1}{|\text{total ham emails (training set)}| + 2}$$

$$\mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0 + 1}{2 + 2} = \frac{1}{4}$$

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$\begin{aligned}
 &= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 &= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}
 \end{aligned}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"you"} \mid \text{ham}) =$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"buy"} \mid \text{ham}) = \frac{0 + 1}{2 + 2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) =$$

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$\begin{aligned}
 & \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) \\
 = & \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 = & \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}
 \end{aligned}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1 + 1}{3 + 2} = \frac{2}{5} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1 + 1}{2 + 2} = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = \frac{0 + 1}{2 + 2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) =$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) =$$

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$\begin{aligned}
 & \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) \\
 = & \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})} \\
 = & \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}
 \end{aligned}$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5}$$

$$\mathbb{P}(\text{"buy"} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3+1}{3+2} = \frac{4}{5}$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5}}{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{5}} \approx 0.7544$$

Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \quad \mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \quad \mathbb{P}(\text{"buy"} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \quad \mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

EXAMPLE $\mathbb{P}(\text{spam} \mid \text{"You buy Valium!"})$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"valium"}\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"} \mid \text{spam}) \mathbb{P}(\text{"buy"} \mid \text{spam}) \mathbb{P}(\text{"valium"} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"} \mid \text{ham}) \mathbb{P}(\text{"buy"} \mid \text{ham}) \mathbb{P}(\text{"valium"} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$= \frac{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5}}{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{5}} \approx 0.7544$$



Email	Label
Buy Valium!	Spam
You good?	Ham
Valium help you.	Spam
Good Valium help.	Spam
I need Valium for my health condition.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5}$$

$$\mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{spam}) = \frac{1 + 1}{3 + 2} = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"} \mid \text{ham}) = \frac{1 + 1}{2 + 2} = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"} \mid \text{spam}) = \frac{1 + 1}{3 + 2} = \frac{2}{5}$$

$$\mathbb{P}(\text{"buy"} \mid \text{ham}) = \frac{0 + 1}{2 + 2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"valium"} \mid \text{spam}) = \frac{3 + 1}{3 + 2} = \frac{4}{5}$$

$$\mathbb{P}(\text{"valium"} \mid \text{ham}) = \frac{1 + 1}{2 + 2} = \frac{1}{2}$$

UNDERFLOW PREVENTION

- Multiplication of many probabilities, each of which will be between 0 and 1, can result in floating-point underflow. The product will be too small and will result in arithmetic underflow.

UNDERFLOW PREVENTION

- Multiplication of many probabilities, each of which will be between 0 and 1, can result in floating-point underflow. The product will be too small and will result in arithmetic underflow.
- Reminder: Log property:

$$\log(xy) = \log(x) + \log(y)$$

UNDERFLOW PREVENTION

- Multiplication of many probabilities, each of which will be between 0 and 1, can result in floating-point underflow. The product will be too small and will result in arithmetic underflow.
- Reminder: Log property:

$$\log(xy) = \log(x) + \log(y)$$

- Summing logs of probabilities is better than multiplying probabilities

$$\begin{aligned}\log\left(\prod_{i=1}^n p_i\right) &= \log(p_1 p_2 \dots p_n) = \log(p_1) + \log(p_2) + \dots + \log(p_n) \\ &= \sum_{i=1}^n \log(p_i)\end{aligned}$$

APPLYING UNDERFLOW PREVENTION

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\})$$

APPLYING UNDERFLOW PREVENTION

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\})$$

$$\iff \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})$$

Denominators are equal and cancel when comparing

APPLYING UNDERFLOW PREVENTION

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\})$$

$$\iff \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})$$

$$\iff \mathbb{P}(w_1 \mid \text{spam}) \mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam}) \mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham}) \mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham}) \mathbb{P}(\text{ham})$$

APPLYING UNDERFLOW PREVENTION

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\}) \approx \frac{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \dots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \dots, w_n\})$$

$$\iff \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{spam}) \mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \dots, w_n\} \mid \text{ham}) \mathbb{P}(\text{ham})$$

$$\iff \mathbb{P}(w_1 \mid \text{spam}) \mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam}) \mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham}) \mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham}) \mathbb{P}(\text{ham})$$

Taking the log of two sides:

$$\iff \log(\mathbb{P}(\text{spam})) + \sum_{i=1}^n \log(\mathbb{P}(w_i \mid \text{spam})) > \log(\mathbb{P}(\text{ham})) + \sum_{i=1}^n \log(\mathbb{P}(w_i \mid \text{ham}))$$

SUMMARY: NAIVE BAYES ALGORITHM STEPS

1. TRAINING

1.1. Compute the proportion of emails in the **training set** that is spam or ham:

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}}$$

$$\mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

1.2. Iterate over the **training set**, for each unique word **x**, count:

- How many **spam emails** in the training set contain **x**
- How many **ham emails** in the training set contain **x**

SUMMARY: NAIVE BAYES ALGORITHM STEPS

1. TRAINING

1.1. Compute the proportion of emails in the **training set** that is spam or ham:

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}}$$

$$\mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

1.2. Iterate over the **training set**, for each unique word **x**, count:

- How many **spam emails** in the training set contain **x**
- How many **ham emails** in the training set contain **x**

2. TESTING

Iterate over the **test set**, for each unlabelled email **D**:

- Create a set **S** of **n** unique words appearing in **D**: $\{w_1, w_2, \dots, w_n\}$
- For each word w_i in set **S**, calculate:

$$\mathbb{P}(w_i | \text{spam}) = \frac{|\text{total spam emails (training set) containing } w_i| + 1}{|\text{total spam emails (training set)}| + 2}$$

$$\mathbb{P}(w_i | \text{ham}) = \frac{|\text{total ham emails (training set) containing } w_i| + 1}{|\text{total ham emails (training set)}| + 2}$$

- Note: If word w_i doesn't appear in the training set, we still calculate the above probabilities, with:

$$|\text{total spam emails (training set) containing } w_i| = |\text{total ham emails (training set) containing } w_i| = 0$$

- If $\log(\mathbb{P}(\text{spam})) + \sum_{i=1}^n \log(\mathbb{P}(w_i | \text{spam})) > \log(\mathbb{P}(\text{ham})) + \sum_{i=1}^n \log(\mathbb{P}(w_i | \text{ham}))$

Predict email **D** as **spam**

Otherwise, predict email **D** as **ham**



ALEX TSUN