CSE 312: Foundations of Computing II Autumn 2024

Problem Set 1

Due: Wednesday, October 2, by 11:59pm

Instructions

Solutions format. Every step in your solution should be explained carefully. The logical reasoning behind your solution should be sound and evident from your write-up.

For example, if you are asked to compute the number of ways to permute the set $\{1, 2, 3, 4\}$ that start with 1 or 2, it is not enough to provide the answer 12. A complete approach would explain that (1) we can count separately the permutations starting with 1 and those starting with 2, and that (2) the two sets are disjoint, and hence the overall number is the sum of the numbers of permutations of each type. Then, (3) explain that there are $3!$ permutations of each type. Finally, (4) say that the overall number totals to $2 \cdot 3! = 12$.

A higher number of mathematical symbols in your solution will not make your solution more precise or "better" – what *is* important is that the logical flow is complete and can be followed by the graders. Relying exclusively on mathematical symbols in fact often make the solution less readable. Avoid expressions such as "it easy to see" and "clearly" – just explain these steps.

Also, you may find the following short note (by Francis E. Su at Harvey Mudd) helpful.

Unless a problem states otherwise, you can leave your answer in terms of factorials, combinations, etc., for instance 26^7 or $26!/7!$ or $26 \cdot \binom{26}{7}$ are all good forms for final answers.

Collaboration policy. You are required to submit your own solutions. You are allowed to discuss the homework with other students. However, the write up must clearly be your own, and moreover, you must be able to explain your solution at any time. We reserve ourselves the right to ask you to explain your work at any time in the course of this class.

Late policy. You have a total of **six** late days during the quarter, but can only use up to three late days on any one problem set. Please plan ahead, as we will not be willing to add any additional late days except in absolute, verifiable emergencies. The final problem set will not be accepted late (however, it will be due only on Friday of the last week of class).

Solutions submission. You must submit your solution via Gradescope. In particular:

- Submit a *single* PDF file containing the solution to all tasks in the homework. Each numbered task should be solved on its own page (or pages). Follow the prompt on Gradescope to link tasks to your pages.
- Do not write your name on the individual pages – Gradescope will handle that.
- We encourage you to typeset your solution. The homepage provides links to resources to help you doing so using LaTeX. If you do use another tool (e.g., Microsoft Word), we request that you use a proper equation editor to display math (MS Word has one). For example, you should be able to write $\sum_{i=1}^{n} x^i$ instead of $\mathrm{x\hat{}1 + x\hat{}2 + \ldots + x\hat{}n}$. You can also provide a handwritten solution, as long as it is on a single PDF file that satisfies the above submission format requirements. It is your responsibility to make sure handwritten solutions are readable – we will *not* grade unreadable write-ups.

Task 1 – Counting Words [30 pts]

We want to count the number of strings of length 4 from the English alphabet $\{A, B, \cdots, Z\}$ subject to a number of different constraints. Note that we consider the English alphabet here to consist of 6 *vowels* ($\{A, E, I, O, U, Y\}$) and 20 *consonants*.

How many strings are there which ...

a) ... are only made of vowels?

b) ... are only made of consonants?

c) ... have *exactly* one vowel?

d) ... have *exactly* two vowels?

e) ... have at most two vowels, which may only appear in the second and fourth position?

f) ... have at least one vowel?

In all cases, explain your reasoning exactly – do not just give numbers or unjustified calculations.

Task 2 – TAs [20 pts]

For a lecture, we need to assign 12 TAs to 6 available sections. How many ways are there to do so, if ...

a) ... the assignment is unrestricted (e.g., some sections may have 0 TAs, some TAs may be assigned to all sections, ...)?

b) ... every section is assigned to exactly two TAs?

c) ... every TA is assigned to exactly one section?

d) ... every section is assigned two TAs and every TA is assigned to exactly one section?

Task 3 – Arrangements [14 pts]

How many different ways are there to arrange the letters in the following words?

a) **MISSISSAUGA**

b) **statistics**

Task 4 – From here to there [20 pts]

In this problems you will consider paths on the integer grid that start at (0,0) in which every step increments one coordinate by 1 and leaves the other unchanged.

a) How many such paths are there from (0,0) to (25,75)?

b) How many such paths are there from (0,0) to (25,75) that go through (10,35)? How many such paths if they must go through (15,40) instead?

c) How many such paths are there from (0,0) to (25,75) that go through (10,35), but do *not* go through (15,40).

d) How many such paths from (0,0) to (25,75) are there that go through neither of (10,35) nor (15,40)?

Task 5 – At least one [16 pts]

In class, given integers $n \geqslant 0$ and $k \geqslant 1$, we were able to count the number of integer solutions $x_1, \ldots, x_k \geqslant 0$ satisfying $\sum_{i=1}^{k} x_i = n$ as $\binom{n+k-1}{k-1}$.

Suppose now that $n \geqslant k \geqslant 1$ and suppose that we want to count the number of integer solutions $y_1, \ldots, y_k \geqslant 1$ such that $\sum_{i=1}^{k} y_i = n$. Show how to use the count above (or similar ideas to the proof of that formula) to write a nice formula for this case.