CSE 312 Foundations of Computing II

Lecture 18: CLT & Polling

Review CDF of normal distribution

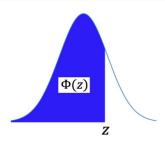
Fact. If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Y = aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

Standard (unit) normal = $\mathcal{N}(0, 1)$

CDF.
$$\Phi(z) = P(Z \le z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} dx$$
 for $Z \sim \mathcal{N}(0, 1)$

Note: $\Phi(z)$ has no closed form – generally given via tables

$\begin{array}{l} \mbox{Review} \\ \mbox{Table of } \Phi(z) \mbox{ CDF of} \\ \mbox{Standard Normal} \\ \mbox{Distribution} \end{array}$



$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0, 1)$										
0.1 0.53983 0.5438 0.54776 0.55172 0.55567 0.55962 0.56356 0.56749 0.57142 0.57355 0.2 0.57926 0.58317 0.58706 0.59095 0.59483 0.59871 0.60257 0.60642 0.61026 0.61409 0.3 0.61791 0.62172 0.62522 0.6293 0.63307 0.63683 0.60428 0.64431 0.64803 0.65173 0.4 0.65542 0.65917 0.66276 0.66644 0.67003 0.67364 0.67724 0.68082 0.64339 0.62757 0.72575 0.72907 0.73237 0.73565 0.73891 0.74215 0.74357 0.74557 0.7575 0.7575 0.78544 0.76115 0.76424 0.7673 0.77337 0.77637 0.77637 0.77935 0.7823 0.88241 0.8 0.78144 0.79138 0.79255 0.80234 0.80511 0.83398 0.83617 0.83891 1.0 0.84134 0.8435 0.8464 <	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
$ 0.61791 0.62172 0.62552 0.6293 0.63307 0.63683 0.64058 0.64431 0.64803 0.65173 \\ 0.65542 0.6591 0.66276 0.6664 0.67003 0.67364 0.67724 0.68082 0.68439 0.68793 \\ 0.69146 0.69497 0.69847 0.70194 0.7054 0.70884 0.71226 0.71566 0.71904 0.7224 \\ 0.6 0.72575 0.72907 0.73327 0.73565 0.73891 0.74215 0.74537 0.74857 0.7515 0.7519 \\ 0.75804 0.76115 0.76424 0.7673 0.77035 0.77337 0.77637 0.77935 0.7823 0.78524 \\ 0.8 0.78814 0.79103 0.79389 0.79673 0.79955 0.80234 0.80511 0.80785 0.81057 0.81327 \\ 0.81594 0.81859 0.82121 0.82381 0.82639 0.82894 0.83147 0.83398 0.83646 0.83891 \\ 1.0 0.84134 0.84375 0.84614 0.84849 0.85083 0.85314 0.85543 0.85769 0.85993 0.86214 \\ 1.1 0.86433 0.8665 0.88867 0.88976 0.87266 0.87493 0.87698 0.879 0.881 0.88298 \\ 1.2 0.8493 0.8866 0.88877 0.89065 0.89251 0.89435 0.89617 0.89796 0.89973 0.90147 \\ 1.3 0.9032 0.9049 0.90658 0.90824 0.90988 0.91149 0.91309 0.91466 0.91621 0.91774 \\ 1.4 0.91924 0.92073 0.9222 0.92364 0.92507 0.92647 0.92785 0.92922 0.93056 0.93189 \\ 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93043 0.94062 0.94179 0.94295 0.94408 \\ 1.6 0.9452 0.9463 0.94738 0.94854 0.94950 0.95053 0.95154 0.95154 0.95254 0.95254 0.94549 \\ 1.7 0.95543 0.95673 0.95728 0.95818 0.95907 0.95994 0.9608 0.96164 0.96246 0.96327 \\ 1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 \\ 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.97414 0.975 0.97558 0.97615 0.9767 \\ 0.97725 0.97778 0.97831 0.9782 0.9732 0.9782 0.9803 0.98077 0.98124 0.98169 \\ 0.91728 0.99845 0.98637 0.98574 0.98377 0.98574 0.98569 \\ 0.9813 0.99845 0.98657 0.99573 0.99732 0.97928 0.9803 0.98077 0.98124 0.98169 \\ 0.997128 0.99763 0.99845 0.98778 0.98809 0.99841 0.99158 \\ 0.99734 0.99845 0.98577 0.99326 0.99935 0.99906 0.99914 0.99613 0.99613 0.99767 0.99857 0.985$	0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.4 0.65542 0.66911 0.66276 0.6664 0.67003 0.67364 0.67724 0.68082 0.68439 0.68793 0.5 0.69146 0.69497 0.69847 0.70194 0.7054 0.70884 0.71226 0.71566 0.71904 0.7224 0.6 0.72575 0.72907 0.73237 0.73565 0.73891 0.74357 0.74857 0.75175 0.75175 0.7549 0.7 0.75804 0.76115 0.76244 0.7673 0.77035 0.77337 0.77935 0.7823 0.7823 0.8 0.78144 0.79133 0.77938 0.77035 0.77337 0.77935 0.8723 0.7823 0.8 0.78144 0.79133 0.79339 0.79735 0.82344 0.80511 0.80785 0.81057 0.81327 0.9 0.81594 0.81859 0.82121 0.82381 0.82639 0.82844 0.85143 0.85769 0.85993 0.86214 1.1 0.86433 0.8665 0.86864 0.87076 0.87286 0.87493 0.8769 0.89793 0.90147 1.3 0.9049 0.90658 0.90824 0.99435 0.89417 0.89796 0.89973 0.90147 1.4 0.9129 0.90493 0.99658 0.99267 0.92785 0.99226 0.93056 0.99146 1.4 0.9129 0.90448 0.93574 0.93292 0.93056 0.99177 0.94295 0.94408 1.5 <td>0.2</td> <td>0.57926</td> <td>0.58317</td> <td>0.58706</td> <td>0.59095</td> <td>0.59483</td> <td>0.59871</td> <td>0.60257</td> <td>0.60642</td> <td>0.61026</td> <td>0.61409</td>	0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.5 0.69146 0.69497 0.69847 0.70194 0.7054 0.70884 0.71226 0.71566 0.71904 0.7224 0.6 0.72575 0.72907 0.73237 0.73565 0.73891 0.74215 0.74537 0.74857 0.75175 0.7549 0.7 0.75804 0.76115 0.76424 0.7673 0.77035 0.77337 0.77637 0.77935 0.7823 0.78524 0.8 0.7814 0.79103 0.79389 0.79673 0.77955 0.80234 0.80511 0.80785 0.81057 0.81327 0.9 0.81594 0.81357 0.84614 0.84849 0.85083 0.85141 0.88769 0.85093 0.86214 1.0 0.84134 0.84575 0.84614 0.87076 0.87286 0.87493 0.87698 0.879 0.8811 0.88298 1.2 0.88493 0.8666 0.8877 0.89065 0.89251 0.89435 0.89617 0.89766 0.89973 0.90147 1.3 0.9027 0.9222 0.92364 0.99088 0.91149 0.91309 0.91466 0.91621 0.91774 1.4 0.91924 0.92073 0.9222 0.92364 0.92067 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.9448 0.93574 0.93943 0.9468 0.94140 0.96246 0.96256 0.96952 0.95728 1.6 0.9452 0.9463 0.94552 $0.$	0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.6 0.72575 0.72907 0.73237 0.73565 0.73891 0.74215 0.74537 0.74857 0.75175 0.75494 0.7 0.75804 0.76115 0.76424 0.7673 0.77035 0.77337 0.77637 0.77935 0.7823 0.7823 0.8 0.78814 0.79103 0.79389 0.79673 0.79955 0.80234 0.80511 0.80785 0.81057 0.81327 0.9 0.81594 0.81859 0.82121 0.82381 0.82694 0.83147 0.83398 0.83646 0.83891 1.0 0.84134 0.84375 0.84614 0.84849 0.85083 0.8514 0.85433 0.85799 0.8829 1.1 0.84433 0.8665 0.86864 0.87076 0.87286 0.87493 0.8796 0.8973 0.90147 1.3 0.9032 0.9049 0.90658 0.99251 0.89435 0.89617 0.8976 0.8973 0.90147 1.4 0.91924 0.92073 0.9222 0.92364 0.9257 0.92647 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94662 0.94179 0.94295 0.94408 1.6 0.9467 0.96435 0.95728 0.99594 0.95053 0.95154 0.95254 0.95352 0.95476 1.9 0.9463 0.94738 0.94845 0.9495 0.95636 0	0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
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1.0 0.84134 0.84375 0.84614 0.84849 0.85083 0.85144 0.85543 0.85769 0.85993 0.86214 1.1 0.86433 0.8665 0.86864 0.87076 0.87286 0.87493 0.8798 0.879 0.881 0.88298 1.2 0.88493 0.86866 0.88877 0.89065 0.89251 0.89435 0.8917 0.89796 0.89973 0.90147 1.3 0.9032 0.9049 0.90658 0.90824 0.90988 0.91149 0.91309 0.91466 0.91621 0.91774 1.4 0.91924 0.92073 0.9222 0.92364 0.92507 0.92647 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94062 0.94179 0.94295 0.94408 1.6 0.9452 0.9463 0.94738 0.94845 0.9455 0.9553 0.95154 0.95254 0.95322 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.95077 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96526 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97672 2.0 0.97725 0.9778 0.97731 0.97827 0.9732 0.97828 0.9803 0.98077 0.98124 0.98577 2.2 0.98616 0.9863	0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
1.1 0.86433 0.8665 0.86844 0.87076 0.87286 0.87493 0.87698 0.879 0.881 0.88298 1.2 0.88493 0.88686 0.88877 0.89065 0.89251 0.89435 0.89617 0.89796 0.89733 0.90147 1.3 0.9032 0.9049 0.90658 0.90824 0.90988 0.91149 0.91309 0.91466 0.91621 0.91774 1.4 0.91924 0.92073 0.9222 0.92364 0.92507 0.92647 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94062 0.94179 0.94295 0.94408 1.6 0.9452 0.9463 0.94738 0.94845 0.9495 0.95053 0.95154 0.95254 0.95352 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.95077 0.95084 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.96856 0.96995 0.97062 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.9741 0.9755 0.97615 0.97675 2.0 0.97775 0.9778 0.97831 0.97842 0.98461 0.9857 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 $0.$	0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
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1.3 0.9032 0.9049 0.90658 0.90824 0.90988 0.91149 0.91309 0.91466 0.91621 0.91774 1.4 0.91924 0.92073 0.9222 0.92364 0.92507 0.92647 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94062 0.94179 0.94295 0.94408 1.6 0.9452 0.9463 0.94738 0.94845 0.9495 0.95053 0.95154 0.95254 0.95252 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.95077 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96455 0.96562 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 1.9 0.97128 0.97133 0.97257 0.9732 0.97381 0.97441 0.975 0.97675 0.97677 2.0 0.97725 0.97778 0.97831 0.97832 0.97932 0.9803 0.98077 0.98124 0.98169 2.1 0.98214 0.98257 0.9833 0.9731 0.97932 0.97982 0.9803 0.98077 0.98124 0.98169 2.2 0.9861 0.98257 0.9833 0.9731 0.97932 0.98422 0.98461 0.9857 0.98577 0.98573 2.3 0.98616 0.98645 $0.$	1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493	0.87698	0.879	0.881	0.88298
1.4 0.91924 0.92073 0.9222 0.92364 0.92507 0.92647 0.92785 0.92922 0.93056 0.93189 1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94062 0.94179 0.94295 0.94408 1.6 0.9452 0.9463 0.94738 0.94845 0.9495 0.95533 0.95154 0.95254 0.95352 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.95907 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.9608 0.96164 0.96246 0.96327 1.8 0.97128 0.97133 0.97257 0.9732 0.97381 0.97674 0.96856 0.96926 0.96995 0.97672 2.0 0.97725 0.97778 0.9731 0.97822 0.97832 0.97832 0.98037 0.98124 0.98169 2.1 0.98214 0.98257 0.97331 0.97822 0.97822 0.98461 0.985 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 0.98745 0.98778 0.98809 0.9884 0.9887 0.98899 2.3 0.98928 0.98956 0.99833 0.99016 0.99966 0.99314 0.99134 0.99158 2.4 0.9918 0.999202 0.99244 0.99245 <t< td=""><td>1.2</td><td>0.88493</td><td>0.88686</td><td>0.88877</td><td>0.89065</td><td>0.89251</td><td>0.89435</td><td>0.89617</td><td>0.89796</td><td>0.89973</td><td>0.90147</td></t<>	1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.5 0.93319 0.93448 0.93574 0.93699 0.93822 0.93943 0.94062 0.94179 0.94295 0.94408 1.6 0.9452 0.9463 0.94738 0.94845 0.9495 0.95053 0.95154 0.95254 0.95352 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.95907 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96522 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.97441 0.975 0.97558 0.97615 0.97677 2.0 0.97725 0.97778 0.97831 0.97822 0.97982 0.9803 0.98077 0.98124 0.98169 2.1 0.98214 0.98257 0.983 0.98311 0.98382 0.98422 0.98461 0.985 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 0.98778 0.98809 0.9884 0.9887 0.98899 2.3 0.98928 0.98956 0.98983 0.9901 0.99036 0.99036 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99355 0.99324 0.99343 0.99361 2.5 0.99379 0.99366 0.99413 0.99433 0	1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.6 0.9452 0.9463 0.94738 0.94845 0.9495 0.95053 0.95154 0.95254 0.95352 0.95342 0.95449 1.7 0.95543 0.95637 0.95728 0.95818 0.9507 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.97441 0.975 0.97558 0.97615 0.9767 2.0 0.97725 0.97778 0.97831 0.97822 0.97982 0.9803 0.98077 0.98124 0.98169 2.1 0.98214 0.98257 0.983 0.98341 0.98382 0.98422 0.98461 0.985 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 0.98778 0.98809 0.9844 0.9857 0.98899 2.3 0.98928 0.98956 0.98933 0.9901 0.99036 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99244 0.99245 0.99266 0.99266 0.99324 0.99343 0.99361 2.5 0.99379 0.99366 0.99413 0.99433 0.99461 0.99477 0.99422 0.99566 0.9952 2.6 0.99553 0.99654 0.99674 0.99673 0.99	1.4			0.9222	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.7 0.95543 0.95637 0.95728 0.95818 0.95907 0.95994 0.9608 0.96164 0.96246 0.96327 1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.97441 0.975 0.97558 0.97615 0.9767 2.0 0.97725 0.97778 0.97831 0.97822 0.97982 0.9803 0.98077 0.98124 0.98169 2.1 0.98214 0.98257 0.983 0.98341 0.98382 0.98422 0.98461 0.985 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 0.98778 0.98809 0.9844 0.9857 0.98574 2.3 0.98928 0.98956 0.98933 0.9901 0.99036 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99244 0.99245 0.99266 0.99266 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.99433 0.99461 0.99477 0.99422 0.99566 0.99522 2.6 0.99534 0.99547 0.99566 0.99573 0.99598 0.99609 0.99621 0.99632 0.99632 2.7 0.99653 0.99674 0.99677 0.99774 0.99788	1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.8 0.96407 0.96485 0.96562 0.96638 0.96712 0.96784 0.96856 0.96926 0.96995 0.97062 1.9 0.97128 0.97193 0.97257 0.9732 0.97381 0.97441 0.975 0.97558 0.97615 0.9767 2.0 0.97725 0.97778 0.97831 0.97822 0.97982 0.9803 0.98077 0.98124 0.98169 2.1 0.98214 0.98257 0.983 0.98341 0.98382 0.98422 0.98461 0.985 0.98537 0.98574 2.2 0.98611 0.98645 0.98679 0.98713 0.98745 0.98778 0.98809 0.9884 0.9887 0.98899 2.3 0.98928 0.98956 0.98833 0.9901 0.99036 0.99061 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99266 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.99446 0.99461 0.99477 0.99422 0.99506 0.99522 2.6 0.99534 0.99674 0.99674 0.99633 0.99702 0.99711 0.99622 0.99632 0.99632 2.7 0.99653 0.99674 0.99767 0.99774 0.99788 0.99795 0.99801 0.99807 2.8 0.99744 0.99813 0.99825 0.99831 0.99836 <t< td=""><td>1.6</td><td>0.9452</td><td>0.9463</td><td>0.94738</td><td>0.94845</td><td>0.9495</td><td>0.95053</td><td>0.95154</td><td>0.95254</td><td>0.95352</td><td>0.95449</td></t<>	1.6	0.9452	0.9463	0.94738	0.94845	0.9495	0.95053	0.95154	0.95254	0.95352	0.95449
1.90.971280.971930.972570.97320.973810.974410.9750.975580.976150.976752.00.977250.977780.978310.978820.979320.979820.98030.980770.981240.981692.10.982140.982570.9830.983410.983820.984220.984610.9850.985370.985742.20.98610.986450.986790.987130.987450.987780.988090.98840.98870.988992.30.989280.989560.989330.99010.990360.990610.990860.991110.991340.991582.40.99180.992020.992240.992450.992660.992860.993050.993240.993430.993612.50.993790.993960.994130.99430.994460.994610.994770.994920.995060.99522.60.995340.995470.99560.995730.995850.995980.996090.996210.996320.996432.70.996530.996640.996740.996830.997020.997110.99720.997280.997362.80.997440.997520.99760.997670.997740.997880.997950.998010.998072.90.998130.998190.998250.998310.998360.998410.998460.998510.998560.99861	1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.9608	0.96164	0.96246	0.96327
2.00.977250.977780.978310.978820.979320.979820.98030.980770.981240.981692.10.982140.982570.9830.983410.983820.984220.984610.9850.985740.985742.20.98610.986450.986790.987130.987450.987780.988090.98840.98870.988992.30.989280.989560.989830.99010.990360.990610.990860.991110.991340.991582.40.99180.992020.992240.992450.992660.992860.993050.993240.993430.993612.50.993790.993960.994130.99430.994460.994610.994770.994920.995060.99522.60.995340.995470.99560.995730.995850.995980.996090.996210.996320.996332.70.996530.996640.996740.996830.997020.997110.99720.997280.997362.80.997440.997520.99760.997740.997810.997880.997950.998010.998072.90.98130.98190.998250.998310.998360.998410.998460.998510.998560.99861	1.8	0.96407	0.96485	0.96562	0.96638		0.96784	0.96856	0.96926	0.96995	0.97062
2.1 0.98214 0.98257 0.983 0.98341 0.98382 0.98422 0.98461 0.985 0.98537 0.98574 2.2 0.9861 0.98645 0.98679 0.98713 0.98745 0.98778 0.98809 0.9884 0.9887 0.98899 2.3 0.98928 0.98956 0.98933 0.9901 0.99036 0.99061 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99266 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.99433 0.99446 0.99461 0.99477 0.99492 0.99506 0.9952 2.6 0.99534 0.99547 0.9956 0.99573 0.99585 0.99598 0.99609 0.99621 0.99632 0.99632 0.99632 0.99738 0.99711 0.9972 0.99728 0.99736 2.7 0.99653 0.99674 0.99683 0.99774 0.99788 0.99725 0.99728 0.99728 0.99728 0.99728 0.99728 0.99736 </td <td>1.9</td> <td></td> <td>0.97193</td> <td>0.97257</td> <td>0.9732</td> <td>0.97381</td> <td>0.97441</td> <td>0.975</td> <td></td> <td>0.97615</td> <td>0.9767</td>	1.9		0.97193	0.97257	0.9732	0.97381	0.97441	0.975		0.97615	0.9767
2.2 0.9861 0.98645 0.98679 0.98713 0.98745 0.98778 0.98809 0.9884 0.9887 0.98899 2.3 0.98928 0.98956 0.98933 0.9901 0.99036 0.99061 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99286 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.99433 0.99446 0.99461 0.99477 0.99492 0.99506 0.9952 2.6 0.99534 0.99547 0.9956 0.99573 0.99585 0.99598 0.99609 0.99621 0.99632 0.99643 2.7 0.99653 0.99674 0.99683 0.99693 0.99702 0.99711 0.99728 0.99736 2.8 0.99744 0.99752 0.9976 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.98813 0.99825 0.99831 0.99836 0.99841 0.99846 0.99851 0.99856 0.99861	2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.9803	0.98077	0.98124	0.98169
2.3 0.98928 0.98956 0.98983 0.9901 0.99036 0.99061 0.99086 0.99111 0.99134 0.99158 2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99286 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.99436 0.99446 0.99461 0.99477 0.99492 0.99506 0.9952 2.6 0.99534 0.99547 0.99566 0.99573 0.99585 0.99509 0.99621 0.99632 0.99643 2.7 0.99653 0.99674 0.99683 0.99693 0.99702 0.99711 0.9972 0.99728 0.99736 2.8 0.99744 0.99752 0.9976 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.99813 0.99819 0.99825 0.99831 0.99836 0.99841 0.99846 0.99851 0.99856 0.99861	2.1	0.98214	0.98257	0.983	0.98341	0.98382	0.98422	0.98461	0.985	0.98537	0.98574
2.4 0.9918 0.99202 0.99224 0.99245 0.99266 0.99286 0.99305 0.99324 0.99343 0.99361 2.5 0.99379 0.99396 0.99413 0.9943 0.99446 0.99461 0.99477 0.99422 0.99506 0.9952 2.6 0.99534 0.99547 0.9956 0.99573 0.99585 0.99598 0.99609 0.99621 0.99632 0.99643 2.7 0.99653 0.99664 0.99674 0.99683 0.99693 0.99711 0.9972 0.99728 0.99736 2.8 0.99744 0.99752 0.9976 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.99813 0.99819 0.99825 0.99831 0.99841 0.99846 0.99851 0.99856 0.99861	2.2		0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.9884	0.9887	0.98899
2.5 0.99379 0.99396 0.99413 0.9943 0.99446 0.99461 0.99477 0.99492 0.99506 0.9952 2.6 0.99534 0.99547 0.9956 0.99573 0.99585 0.99598 0.99609 0.99621 0.99632 0.99643 2.7 0.99653 0.99664 0.99674 0.99683 0.99693 0.99702 0.99711 0.9972 0.99728 0.99736 2.8 0.99744 0.99752 0.9976 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.99813 0.99819 0.99825 0.99831 0.99841 0.99846 0.99851 0.99856 0.99861	2.3	0.98928	0.98956	0.98983	0.9901	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.6 0.99534 0.99547 0.99566 0.99573 0.99585 0.99598 0.99609 0.99621 0.99632 0.99633 2.7 0.99653 0.99664 0.99674 0.99683 0.99693 0.99702 0.99711 0.99728 0.99736 2.8 0.99744 0.99752 0.9976 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.99813 0.99819 0.99825 0.99831 0.99836 0.99841 0.99851 0.99856 0.99861				0.99224		0.99266					0.99361
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2.8 0.99744 0.99752 0.9976 0.99767 0.99774 0.99781 0.99788 0.99795 0.99801 0.99807 2.9 0.99813 0.99819 0.99825 0.99831 0.99836 0.99841 0.99846 0.99851 0.99861 0.99861		0.99534	0.99547	0.9956	0.99573	0.99585	0.99598	0.99609	0.99621		0.99643
2.9 0.99813 0.99819 0.99825 0.99831 0.99836 0.99841 0.99846 0.99851 0.99856 0.99861			0.99664	0.99674	0.99683	0.99693	0.99702	0.99711		0.99728	0.99736
		0.99744	0.99752	0.9976	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
3.0 0.99865 0.99869 0.99874 0.99878 0.99882 0.99886 0.99889 0.99893 0.99896 0.999	2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
	3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999

 Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0, 1)$

Review Analyzing non-standard normal in terms of $\mathcal{N}(0, 1)$

If
$$X \sim \mathcal{N}(\mu, \sigma^2)$$
, then $\frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$

Therefore,

$$F_X(z) = P(X \le z) = P\left(\frac{X-\mu}{\sigma} \le \frac{z-\mu}{\sigma}\right) = \Phi\left(\frac{z-\mu}{\sigma}\right)$$

Review How Many Standard Deviations Away?

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. $P(|X - \mu| < k\sigma) = P\left(\frac{|X - \mu|}{\sigma} < k\right) =$ $= P\left(-k < \frac{X - \mu}{\sigma} < k\right) = \Phi(k) - \Phi(-k)$

e.g. k = 1: 68% k = 2: 95% k = 3: 99%

Review Central Limit Theorem

 X_1, \ldots, X_n i.i.d., each with expectation μ and variance σ^2

Define
$$S_n = X_1 + \dots + X_n$$
 and $\overline{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $Y_n = \frac{S_n - n\mu}{\sigma \sqrt{n}}$

mean

variance

CLT:

Review Central Limit Theorem

$$Y_n = \frac{X_1 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$$

Theorem. (Central Limit Theorem) The CDF of Y_n converges to the CDF of the standard normal $\mathcal{N}(0,1)$, i.e.,

$$\lim_{n \to \infty} P(Y_n \le y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} dx$$

Also stated as:

- $\lim_{n\to\infty} Y_n \to \mathcal{N}(0,1)$
- $\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$ for $\mu = \mathbb{E}[X_i]$ and $\sigma^2 = \operatorname{Var}(X_i)$

Agenda

• Central Limit Theorem (CLT) Review

• Polling 🔳

Magic Mushrooms

Suppose conducting a poll as to whether to legalize the therapeutic use of "magic mushrooms" prior to vote.

Poll to determine the fraction p of the population expected to vote in favor.

- Call up a random sample of *n* people to ask their opinion
- Report the empirical fraction

Questions

- Is this a good estimate?
- How to choose *n*?



Polling Accuracy

Often see claims that say

"Our poll found 80% support. This poll is accurate to within 5% with 98% probability"

Will unpack what this and how they sample enough people to know this is true.

* When it is 95% this is sometimes written as "19 times out of 20"

Formalizing Polls

Population size N, true fraction of voting in favor p, sample size n. **Problem:** We don't know p, want to estimate it

Polling Procedure

for i = 1, ..., n:

- 1. Pick uniformly random person to call (prob: 1/N)
- 2. Ask them how they will vote

$$X_i = \begin{cases} 1 \\ 0 \end{cases}$$

voting in favor otherwise

Report our estimate of *p*:

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

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Formalizing Polls

Population size *N*, true fraction of voting in favor *p*, sample size *n*. **Problem:** We don't know *p*

Polling Procedure

for i = 1, ..., n:

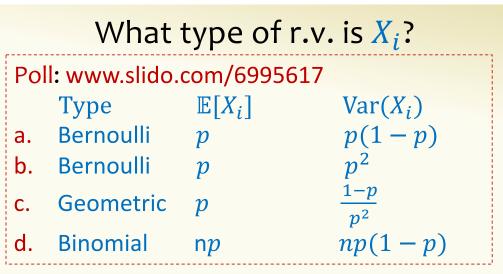
- 1. Pick uniformly random person to call (prob: 1/N)
- 2. Ask them how they will vote

$$X_i = \begin{cases} 1, \\ 0, \end{cases}$$

voting in favor otherwise

Report our estimate of *p*:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$



Random Variables

What type of r.v. is X_i ?

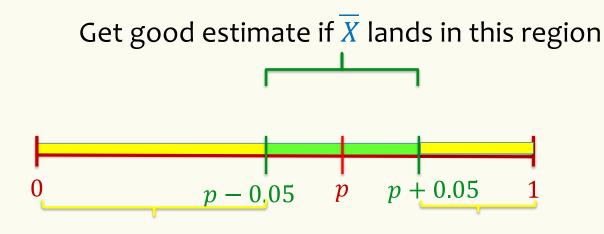
	Туре	$\mathbb{E}[X_i]$	$Var(X_i)$
a.	Bernoulli	p	p(1-p)
b.	Bernoulli	p	p^2
с.	Geometric	p	$\frac{1-p}{p^2}$
d.	Binomial	n p	np(1-p)

What about
$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$
?

Ро	Poll: www.slido.com/6995617									
	$\mathbb{E}[\overline{X}]$	$Var(\overline{X})$								
а.	np	np(1-p)								
b.	p	p(1-p)								
с.	p	p(1-p)/n								
d.	p/n	p(1-p)/n								

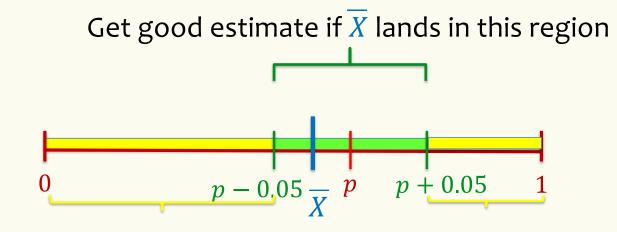
Roadmap: Bounding Error

Goal: Find the value of *n* such that 98% of the time, the estimate \overline{X} is within 5% of the true *p*



Roadmap: Bounding Error

Goal: Find the value of *n* such that 98% of the time, the estimate \overline{X} is within 5% of the true *p*



Question: for what *n* is $P(|\overline{X} - p| > 0.05) \le 0.02$

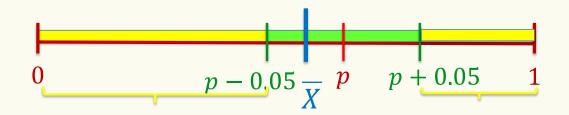
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Central Limit Theorem With i.i.d random variables $X_1, X_2, ..., X_n$ where $\mathbb{E}[X_i] = \mu$ and $\operatorname{Var}(X_i) = \sigma^2$ www.slido.com/6995617 Poll: In the limit \overline{X} is...? a. $\mathcal{N}(0, 1)$ b. $\mathcal{N}(p, p(1-p))$ c. $\mathcal{N}(p, p(1-p)/n)$ d. I don't know

As $n \to \infty$,

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$





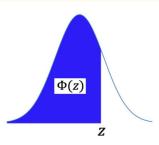
Question: for what *n* is $P(|\overline{X} - p| > 0.05) \le 0.02$

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Table of $\Phi(z)$

Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0, 1)$										
\overline{z}	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884	0.71226	0.71566	0.71904	0.7224
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.7549
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337	0.77637	0.77935	0.7823	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493	0.87698	0.879	0.881	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.9452	0.9463	0.94738	0.94845	0.9495	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.9608	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.9732	0.97381	0.97441	0.975	0.97558	0.97615	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.9803	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.983	0.98341	0.98382	0.98422	0.98461	0.985	0.98537	0.98574
2.2	0.9861	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.9884	0.9887	0.98899
2.3	0.98928	0.98956	0.98983	0.9901	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.9918	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.9943	0.99446	0.99461	0.99477	0.99492	0.99506	0.9952
2.6	0.99534	0.99547	0.9956	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.9972	0.99728	0.99736
2.8	0.99744	0.99752	0.9976	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999



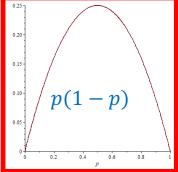
Recap I

Goal: Find the value of *n* such that 98% of the time, the estimate \overline{X} is within 5% of the true *p*

- 1. Define question. For what *n* is $P(|\overline{X} p| > 0.05) \le 0.02$
- 2. Apply CLT: By CLT $\overline{X} \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$
- 3. Convert to a standard normal. Specifically, define $Z = \frac{\overline{X} \mu}{\sigma} = \frac{\overline{X} p}{\sigma}$. Then, by the CLT $Z \to \mathcal{N}(0, 1)$
- 4. Solve for *n*

Recap II

1. For what *n* is $P(|\overline{X} - p| > 0.05) \le 0.02$



2. By CLT $\overline{X} \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$

3. Define
$$Z = \frac{\overline{X} - \mu}{\sigma} = \frac{\overline{X} - p}{\sigma}$$
. Then, by the CLT $Z \to \mathcal{N}(0, 1)$



 $P(|\overline{X} - p| > 0.05) = P(|Z| \cdot \sigma > 0.05)$

Q: Why "≤"? A: This condition on Z is easier to satisfy $= P(|Z| > 0.05/\sigma) = P(|Z| > 0.05\sqrt{n})$ $\leq P(|Z| > 0.1\sqrt{n})$

Recap III

1. Want $P(|\overline{X} - p| > 0.05) \le 0.02$ 2. By CLT $\overline{X} \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1 - p)/n$

3. Define
$$Z = \frac{\overline{X} - \mu}{\sigma} = \frac{\overline{X} - p}{\sigma}$$
. Then, by the CLT $Z \to \mathcal{N}(0, 1)$

$$\frac{1}{\sqrt{p(1-p)}} \text{ is always} \geq 2$$

1. Want $P(|\overline{X} - p| > 0.05) \le 0.02$

2. By CLT $X \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$

$$P(|\overline{X} - p| > 0.05) = P(|Z| \cdot \sigma > 0.05)$$

$$\frac{-P(|Z| > 0.05/\sigma) - P(|Z| > 0.05}{\sqrt{p(1-p)}}$$

Want to choose *n* so that this is at most 0.02
$$\leq P(|Z| > 0.1\sqrt{n})$$

Recap IV

Solve for *n* such that $P(|Z| > 0.1\sqrt{n}) \le 0.02$ where $Z \to \mathcal{N}(0, 1)$

• We assumed *n* is large enough that $Z \sim \mathcal{N}(0, 1)$

Recap V

We want $P(|Z| > 0.1\sqrt{n}) \le 0.02$ where $Z \to \mathcal{N}(0, 1)$

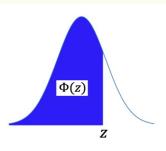
- If we actually had $Z \sim \mathcal{N}(0, 1)$ then enough to show that $P(Z > 0.1\sqrt{n}) \leq 0.01$ since $\mathcal{N}(0, 1)$ is symmetric about 0
- Use $P(Z > z) = 1 \Phi(z)$ where $\Phi(z)$ is the CDF of the Standard Normal Distribution
- Choose *n* so that $0.1\sqrt{n} \ge z$ where $\Phi(z) \ge 0.99$

Recap VI

Table of $\Phi(z)$ CDF of Standard Normal Distribution

Choose *n* so $0.1\sqrt{n} \ge z$ where $\Phi(z) \ge 0.99$

From table z = 2.33 works



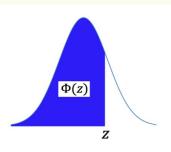
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2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.9972	0.99728	0.99736
2.8	0.99744	0.99752	0.9976	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999

 Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0, 1)$

Recap VII

Choose *n* so $0.1\sqrt{n} \ge z$ where $\Phi(z) \ge 0.99$

From table z = 2.33 works



- So we can choose $0.1\sqrt{n} \ge 2.33$ or $\sqrt{n} \ge 23.3$
- Then $n \ge 543 \ge (23.3)^2$ would be good enough ... if we had $Z \sim \mathcal{N}(0, 1)$
- We only have $Z \rightarrow \mathcal{N}(0, 1)$ so there is some loss due to approximation error.
- Maybe instead consider z = 3.0 with $\Phi(z) \ge 0.99865$ and $n \ge 30^2 = 900$ to cover any loss.

We found an approximate``confidence interval"

We are trying to estimate some parameter (e.g. p). We output an estimator \overline{X} such that $P(|\overline{X} - p| > \epsilon) \le \delta$ for some (ϵ, δ) .

- Often found using CLT
- We say that we are (1δ) *100% confident that the result of our poll (\overline{X}) is an accurate estimate of p to within ϵ *100% percent.
- In our example, ($\epsilon = 0.05, \delta = 0.02$).

Idealized Polling

So far, we have been discussing "idealized polling". Real life is normally not so nice ⊗

Assumed we can sample people uniformly at random, not really possible in practice

- Not everyone responds
- Response rates might differ in different groups
- Will people respond truthfully?

Makes polling in real life much more complex than this idealized model!