CSE 312

Foundations of Computing II

Lecture 18: CLT & Polling

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Review CDF of normal distribution

Fact. If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Y = aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

Standard (unit) normal = $\mathcal{N}(0, 1)$

CDF. $\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} \mathrm{d}x$ for $Z \sim \mathcal{N}(0, 1)$

Note: $\Phi(z)$ has no closed form – generally given via tables

Review

Table of $\Phi(z)$ CDF of Standard Normal Distribution

Φ Table: $\mathbb{P}(Z \le z)$ when $Z \sim \mathcal{N}(0,1)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884	0.71226	0.71566	0.71904	0.7224
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.7549
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337	0.77637	0.77935	0.7823	0.78524
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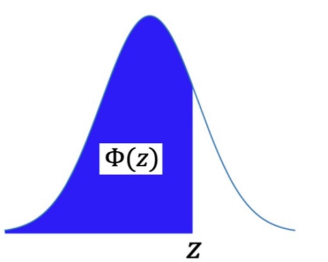
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999



Review Analyzing non-standard normal in terms of $\mathcal{N}(0, 1)$

If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim \mathcal{N}(0, 1)$

Therefore,

$$F_X(z) = P(X \le z) = P\left(\frac{X-\mu}{\sigma} \le \frac{z-\mu}{\sigma}\right) = \Phi\left(\frac{z-\mu}{\sigma}\right)$$

Review How Many Standard Deviations Away?

Let $X \sim \mathcal{N}(\mu, \sigma^2)$.

$$P(|X-\mu| < k\sigma) = P\left(\frac{|X-\mu|}{\sigma} < k\right) =$$

$$= P\left(-k < \frac{X-\mu}{\sigma} < k\right) = \Phi(k) - \Phi(-k)$$

e.g. $k = 1$: 68%
$k = 2$: 95%
$k = 3$: 99%

$E(X+Y) = E(X) + E(Y)$

$Var(X+Y) = Var(X) + Var(Y)$

$E(aX) = aE(X)$

$Var(aX) = a^2 Var(X)$

Review Central Limit Theorem

$X_1, \dots, X_n$ i.i.d., each with expectation μ and variance σ^2

indep.

Define $\boldsymbol{S_n = X_1 + \cdots + X_n}$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. and $Y_n = \frac{S_n - n\mu}{\sigma\sqrt{n}}$ $= \frac{S_n - E(S_n)}{\sigma(S_n)}$

$\bar{X} = \frac{S_n}{n}$

mean $n\mu$ $\boxed{\mu}$ 0

variance $n\sigma^2$ 1

$Var(\bar{X}) = Var(\frac{1}{n} S_n)$

$= \frac{1}{n^2} Var(S_n)$

$= \frac{1}{n^2} \cdot n\sigma^2$

$= \boxed{\frac{\sigma^2}{n}}$

CLT: S_n behaves like $N(n\mu, n\sigma^2)$

$\bar{X} \xrightarrow[n\to\infty]{} N(\mu, \frac{\sigma^2}{n})$

$Y_n \xrightarrow[n\to\infty]{} N(0,1)$

Review Central Limit Theorem

$$Y_n = \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$$

Theorem. (Central Limit Theorem) The CDF of Y_n converges to the CDF of the standard normal $\mathcal{N}(0,1)$, i.e.,

$$\lim_{n\to\infty} P(Y_n \le y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} \mathrm{d}x$$

Also stated as:

- $\lim_{n\to\infty} Y_n \to \mathcal{N}(0,1)$
- $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$ for $\mu = \mathbb{E}[X_i]$ and $\sigma^2 = \mathrm{Var}(X_i)$

Agenda

- Central Limit Theorem (CLT) Review
- Polling

CLT application

- Transmitting a signal. 100 sources independently add noise to signal, each Unif(-1,1). If |total noise| > 10, signal is corrupted.
- Use the CLT to estimate the approximate probability that the signal is not corrupted.

See section 8 worksheet

Magic Mushrooms

Suppose conducting a poll as to whether to legalize the therapeutic use of “magic mushrooms” prior to vote.

Poll to determine the fraction p of the population expected to vote in favor.

- Call up a random sample of n people to ask their opinion
- Report the empirical fraction

Questions

- Is this a good estimate?
- How to choose n?

N

pN

$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n$

$\frac{1}{n}\sum_{i=1}^{n} X_i$

1 0 1 1 0 500

estimate p is $\frac{107}{500}$

Polling Accuracy

Often see claims that say

p.

"Our poll found 80% support. This poll is accurate to within 5% with 98% probability"*

Will unpack what this and how they sample enough people to know this is true.

* When it is 95% this is sometimes written as "19 times out of 20"

Formalizing Polls

Population size N, true fraction of voting in favor p, sample size n.

Problem: We don't know p, want to estimate it

Polling Procedure

for $i = 1, \dots, n$:

1. Pick uniformly random person to call (prob: $1/N$)
2. Ask them how they will vote

$$X_i = \begin{cases} 1, & \text{voting in favor} \\ 0, & \text{otherwise} \end{cases}$$

Report our estimate of p: $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

Formalizing Polls

Population size N, true fraction of voting in favor p, sample size n.

Problem: We don't know p

What type of r.v. is X_i?

Poll: www.slido.com/6995617

	Type	$\mathbb{E}[X_i]$	$\mathrm{Var}(X_i)$
a.	Bernoulli	p	$p(1-p)$
b.	Bernoulli	p	p^2
c.	Geometric	p	$\frac{1-p}{p^2}$
d.	Binomial	np	$np(1-p)$

Polling Procedure

for $i = 1, \ldots, n$:

1. Pick uniformly random person to call (prob: $1/N$)
2. Ask them how they will vote

$$X_i = \begin{cases} 1, & \text{voting in favor} \\ 0, & \text{otherwise} \end{cases}$$

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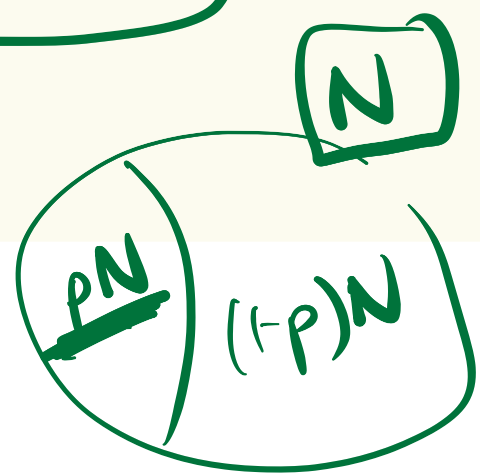


Random Variables

What type of r.v. is X_i?

	Type	$\mathbb{E}[X_i]$	$\mathrm{Var}(X_i)$
a.	Bernoulli	p	$p(1-p)$
b.	Bernoulli	p	p^2
c.	Geometric	p	$\frac{1-p}{p^2}$
d.	Binomial	np	$np(1-p)$

What about $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$?

empirical mean
sample

Poll: www.slido.com/6995617

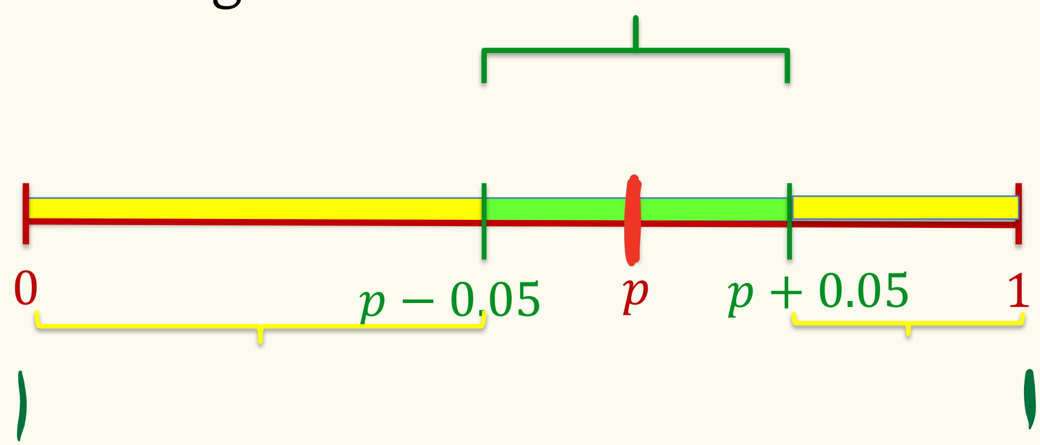
	$\mathbb{E}[\bar{X}]$	$\mathrm{Var}(\bar{X})$
a.	np	$np(1-p)$
b.	p	$p(1-p)$
c.	p	$p(1-p)/n$
d.	p/n	$p(1-p)/n$

Roadmap: Bounding Error

How many people do I need to sample

Goal: Find the value of n such that 98% of the time, the estimate $\bar{X}$ is within 5% of the true p

Get good estimate if $\bar{X}$ lands in this region



Central Limit Theorem

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Poll: In the limit $\overline{X}$ is…?

a. $\mathcal{N}(0, 1)$
b. $\mathcal{N}(p, p(1-p))$
c. $\mathcal{N}(p, p(1-p)/n)$
d. I don't know

With i.i.d random variables $X_1, X_2, \ldots, X_n$ where $\mathbb{E}[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2$

As $n \to \infty$,

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

$\overline{X}$ has mean p, Var $\boxed{\frac{p(1-p)}{n}}$

$\overline{X}$

Roadmap: Bounding Error

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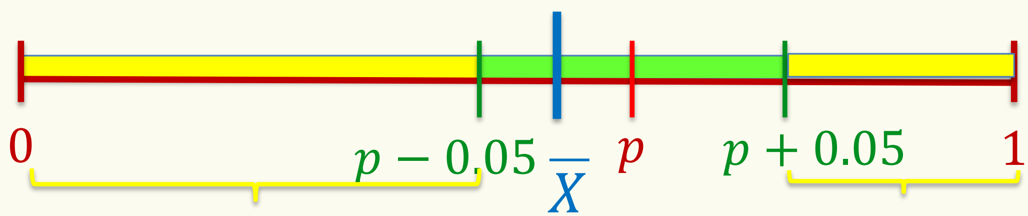


Question: for what n is $\underbrace{P\left(\left|\bar{X} - p\right| > 0.05\right)}_{\text{event that } \bar{X} \text{ is not within 5\% of } p.} \leq 0.02$

$$\bar{X} \to N\left(p, \frac{p(1-p)}{n}\right)$$

$$\boxed{\frac{\bar{X} - p}{\sqrt{\frac{p(1-p)}{n}}} \to N(0,1)}$$

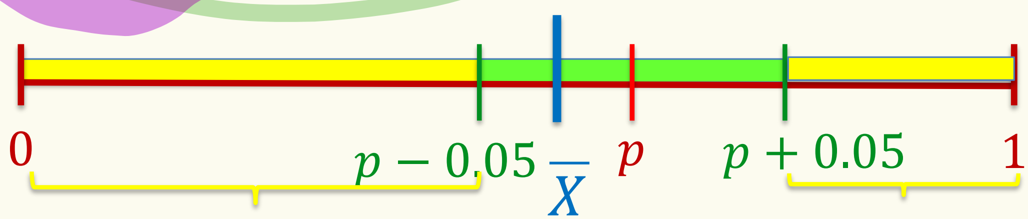
Roadmap: Bounding Error

$$\Rightarrow P\left(|\bar{X} - p| > 0.05\right)$$

$$= P\left(\frac{|\bar{X} - p|}{\sqrt{\frac{p(1-p)}{n}}} > \frac{0.05}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$\sim N(0,1)$

Z



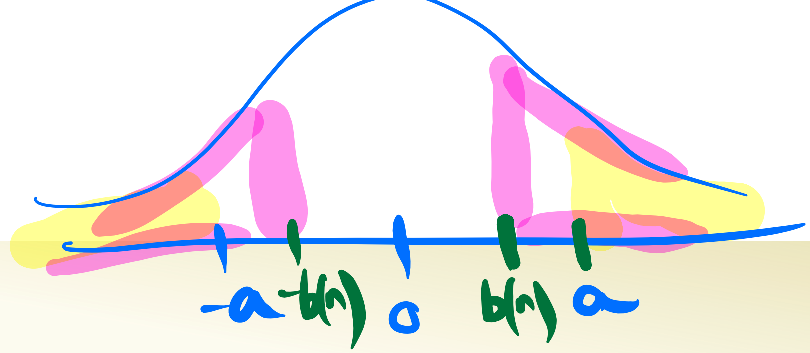
Question: for what n is $P\left(|\bar{X} - p| > 0.05\right) \leq 0.02$

$N(0,1)$

d

$$P\left(|Z| > \frac{0.05}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

a



we don't know p

our goal is to choose n

this prob ≤ 0.02

we use the fact that

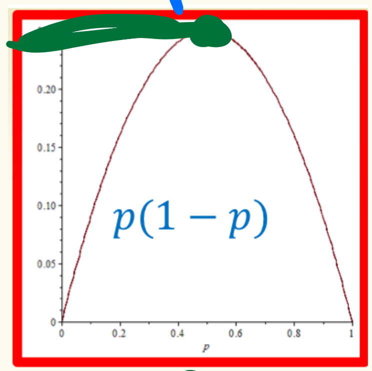
$p(1-p) \leq \frac{1}{4}$

$$a = \frac{0.05\sqrt{n}}{\sqrt{p(1-p)}} \geq \frac{0.05\sqrt{n}}{\sqrt{\frac{1}{4}}} \to b(n)$$

$\forall p$ using

$$\boxed{p(1-p) \leq \frac{1}{4}}$$

$\forall p \in [0,1]$



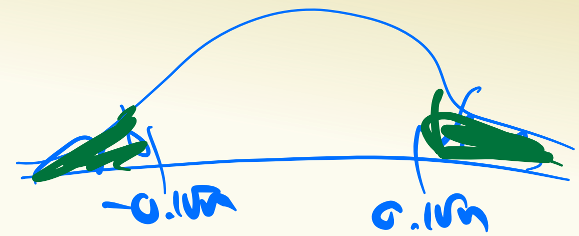
$$0.02 > P(|Z| > b) \geq \boxed{P(|Z| > a)}$$

chosen n

$$\boxed{P\left(|Z| > \frac{0.05\sqrt{n}}{\sqrt{\frac{1}{4}}}\right) \leq 0.02}$$

find n s.t. $\uparrow$ holds

$$\frac{0.05\sqrt{n}}{\sqrt{\frac{1}{4}}} = \frac{0.05\sqrt{n}}{\frac{1}{2}} = 0.1\sqrt{n}$$



find n s.t. $\boxed{P(|Z| > 0.1\sqrt{n})} \le 0.02$

$$= P(Z > 0.1\sqrt{n}) + P(Z \le -0.1\sqrt{n})$$

$$= 2P(Z > 0.1\sqrt{n})$$

$$= 2(1 - P(Z \le 0.1\sqrt{n}))$$

want $\quad 2(1 - \underline{P(Z \le 0.1\sqrt{n})}) \le 0.02$

$$1 - \underline{P(Z \le 0.1\sqrt{n})} \le 0.01$$

$$P(Z \le 0.1\sqrt{n}) \ge 0.99$$

$P(Z \le 0.1\sqrt{n}) \ge 0.99$

Table of $\Phi(z)$

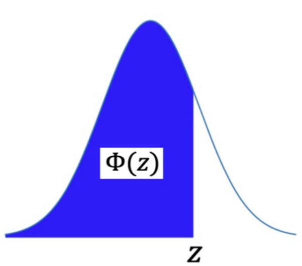
$P(Z \le 2.33) \ge 0.99$

$0.1\sqrt{n} \ge 2.33$

$\sqrt{n} \ge 23.3$

$n \ge (23.3)^2$

≈ 543



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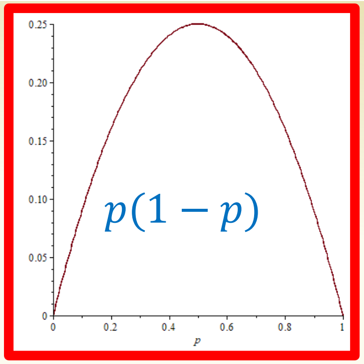
Recap I

Goal: Find the value of n such that 98% of the time, the estimate $\bar{X}$ is within 5% of the true p

1. Define question. For what n is $P\left(\left|\bar{X} - p\right| > 0.05\right) \leq 0.02$
2. Apply CLT: By CLT $\bar{X} \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$
3. Convert to a standard normal. Specifically, define $Z = \frac{\bar{X}-\mu}{\sigma} = \frac{\bar{X}-p}{\sigma}$. Then, by the CLT $Z \to \mathcal{N}(0,1)$
4. Solve for n

Recap II



1. For what n is $P\left(\left|\overline{X} - p\right| > 0.05\right) \leq 0.02$

2. By CLT $\overline{X} \rightarrow \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$

3. Define $Z = \frac{\overline{X}-\mu}{\sigma} = \frac{\overline{X}-p}{\sigma}$. Then, by the CLT $Z \rightarrow \mathcal{N}(0,1)$

$\frac{\mathbf{1}}{\sqrt{\boldsymbol{p(1-p)}}}$ is always $\geq \mathbf{2}$

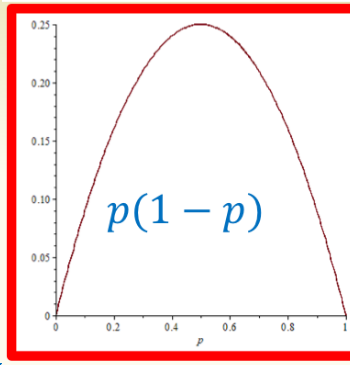
$$P\left(\left|\overline{X} - p\right| > 0.05\right) = P(|Z| \cdot \sigma > 0.05)$$

$$= P(|Z| > 0.05/\sigma) = P\left(|Z| > 0.05\frac{\sqrt{n}}{\sqrt{p(1-p)}}\right)$$

$$\leq P(|Z| > 0.1\sqrt{n})$$

Q: Why "≤"?
A: This condition on Z is easier to satisfy

Recap III



1. Want $P\left(\left|\overline{X}-p\right|>0.05\right)\leq 0.02$
2. By CLT $\overline{X}\rightarrow\mathcal{N}(\mu,\sigma^2)$ where $\mu=p$ and $\sigma^2=p(1-p)/n$
3. Define $Z=\frac{\overline{X}-\mu}{\sigma}=\frac{\overline{X}-p}{\sigma}$. Then, by the CLT $Z\rightarrow\mathcal{N}(0,1)$

$\frac{\mathbf{1}}{\sqrt{\boldsymbol{p}(\mathbf{1}-\boldsymbol{p})}}$ is always $\geq \mathbf{2}$

$$P\left(\left|\overline{X}-p\right|>0.05\right)=P(|Z|\cdot\sigma>0.05)$$

$$=P(|Z|>0.05/\sigma)=P(|Z|>0.05\frac{\sqrt{n}}{\sqrt{p(1-p)}})$$

Want to choose n so that this is at most 0.02

$$\leq P(|Z|>0.1\sqrt{n})$$

Recap IV

Solve for n such that $P(|Z| > 0.1\sqrt{n}) \leq 0.02$ *where* $Z \rightarrow \mathcal{N}(0,1)$

- We assumed n is large enough that $Z \sim \mathcal{N}(0,1)$

Recap V

We want $P(|Z| > 0.1\sqrt{n}) \leq 0.02$ *where* $Z \to \mathcal{N}(0,1)$

- If we actually had $Z \sim \mathcal{N}(0,1)$ then enough to show that $P(Z > 0.1\sqrt{n}) \leq 0.01$ since $\mathcal{N}(0,1)$ is symmetric about 0

- Use $P(Z > z) = 1 - \Phi(z)$ where $\Phi(z)$ is the CDF of the Standard Normal Distribution

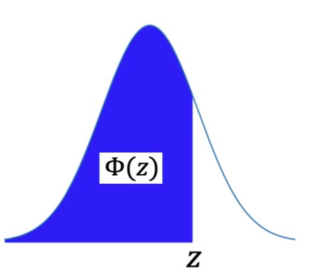
- Choose n so that $0.1\sqrt{n} \geq z$ where $\Phi(z) \geq 0.99$

Recap VI

Table of $\Phi(z)$ CDF of Standard Normal Distribution

Choose n so
$0.1\sqrt{n} \geq z$ where
$\Phi(z) \geq 0.99$

From table $z = 2.33$ works



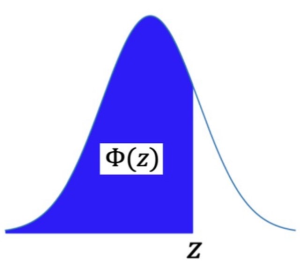
Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0,1)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884	0.71226	0.71566	0.71904	0.7224
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.7549
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337	0.77637	0.77935	0.7823	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493	0.87698	0.879	0.881	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.9452	0.9463	0.94738	0.94845	0.9495	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.9608	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.9732	0.97381	0.97441	0.975	0.97558	0.97615	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.9803	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.983	0.98341	0.98382	0.98422	0.98461	0.985	0.98537	0.98574
2.2	0.9861	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.9884	0.9887	0.98899
2.3	0.98928	0.98956	0.98983	0.9901	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.9918	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.9943	0.99446	0.99461	0.99477	0.99492	0.99506	0.9952
2.6	0.99534	0.99547	0.9956	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.9972	0.99728	0.99736
2.8	0.99744	0.99752	0.9976	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999

Recap VII

Choose n so
$0.1\sqrt{n} \geq z$ where
$\Phi(z) \geq 0.99$

From table $z = 2.33$ works



- So we can choose $0.1\sqrt{n} \geq 2.33$
  or $\sqrt{n} \geq 23.3$
- Then $n \geq 543 \geq (23.3)^2$ would be good enough … if we had $Z \sim \mathcal{N}(0,1)$
- We only have $Z \to \mathcal{N}(0,1)$ so there is some loss due to approximation error.
- Maybe instead consider $z = 3.0$ with $\Phi(z) \geq 0.99865$ and $n \geq 30^2 = 900$ to cover any loss.

We found an approximate``confidence interval”

We are trying to estimate some parameter (e.g. p). We output an estimator $\overline{X}$ such that $P\left(\left|\overline{X}-p\right|>\epsilon\right)\leq\delta$ for some (ϵ,δ).

- Often found using CLT
- We say that we are $(1-\delta)$*100% confident that the result of our poll ($\overline{X}$) is an accurate estimate of p to within ϵ*100% percent.
- In our example, $(\epsilon=0.05,\delta=0.02)$.

Idealized Polling

So far, we have been discussing "idealized polling". Real life is normally not so nice ☹

Assumed we can sample people uniformly at random, not really possible in practice

- Not everyone responds
- Response rates might differ in different groups
- Will people respond truthfully?

Makes polling in real life much more complex than this idealized model!