CSE 312 Section 1

Welcome :)

Turn to your neighbor and introduce yourself:

- Name and pronouns
- Year
- What other classes are you taking this quarter
- Something fun you did over winter break

Awesome resources

- Textbook
- Section problems
- Definitions and Theorems Sheet
- EdStem/OH (to not only ask your own questions but hear others)

Review

- Sum Rule: no overlap in outcomes
- Product Rule: counting choices in stages
- Permutation: Ordering of ***N*** Distinct Objects: ***N!***
- k-Permutation: Ordering ***k*** of ***n*** Distinct Objects: P(n, k) $$\frac{n!}{(n-k)!}$$

- k-Combination: Choosing ***k*** of ***n*** Distinct Objects (order does not matter) : C(n ,k) $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Shout out to William Howard-Snyder and Mitchell Estberg for this slide!

Review

- Complementary counting: **total - opposite**
- Binomial Theorem: $\forall x, y \in \mathbb{R}, \forall n \in \mathbb{N}$: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

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Question 3: "Seating"

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

(a) . . . all couples are to get adjacent seats?

(b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

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Answer: $5! * 2^5$ (By Product Rule)

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Let’s break it down!

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

(b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

- The number of ways to arrange 10 people in 10 seats without any restrictions is: **10!**

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

(b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

- The number of ways to arrange 10 people in 10 seats without any restrictions is: **10!**
- Then, we can treat that couple as a "ninth unit" added to the other 8 individuals, and then there are 2! ways to arrange that two people within their unit. Thus, the number of ways to rearrange people so that that couple will sit in adjacent seats is: **9! * 2!**

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- The number of ways to arrange 10 people in 10 seats without any restrictions is: **10!**
- Then, we can treat that couple as a “ninth unit” added to the other 8 individuals, and then there are 2! ways to arrange that two people within their unit. Thus, the number of ways to rearrange people so that that couple will sit in adjacent seats is: **9! * 2!**
- Finally, by using the method of “Complements”, we ge $10! - 9! * 2 = 8 * 9!$

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

(b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

Alternatively:

- Name the two people in the couple A and B. There are two cases:
  - A can sit on one of the ends, or not. If A sits on an end seat, A has 2 choices and B has 8 possible seats.
  - If A doesn't sit on the end, A has 8 choices and B only has 7.
- So there are a total of $2 \cdot 8 + 8 \cdot 7$ ways A and B can sit. Once they do, the other 8 people can sit in 8! ways since there are no other restrictions.

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Answer:

$$(2 * 8 + 8 * 7)8! = 9 * 8 * 8! = 8 * 9!$$

Question 6 : “Escape the Professor”

There are 6 security professors and 7 theory professors taking part in an escape room. If 4 security professors and 4 theory professors are chosen and paired off, how many pairings (security-theory) are possible?

- Number of ways to choose 4 security professors from 6 $\binom{6}{4}$
- Number of ways to choose 4 theory professors from 7 $\binom{7}{4}$
- Then assign each theory professor to a security professor (4 choices for the first, 3 for the second and so on)
- With the product rule, we get $\boxed{\binom{6}{4}\binom{7}{4}4!}$

Question 9: “Photographs”

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

Question 9: “Photographs”

8 people line up for a picture - how many ways to have <2 people between you and your friend?

There are 2 possible ways to have <2 people between you and your friend: case 1, there are 0 people in between, and case 2, there is exactly 1 person in between.

Since these 2 cases have no overlap, we can use the sum rule.

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8 people line up for a picture - how many ways to have <2 people between you and your friend?

Case 1, there are 0 people in between you and your friend.

Then, we have 7 units to arrange: the 6 other people + 1 unit of you and your friend together. There are 7! ways to arrange 7 distinct units.

Then, within the unit of you and your friend, there are 2! possible arrangements.

This gives a total of 7! * 2 arrangements for case 1.

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Case 2, there is exactly 1 person in between you and your friend.

First, choose the person in between: there are 6 possible choices.
Then, consider you, your friend and that person as a unit. There are 6 distinct units to arrange, in 6! possible ways.
Finally, within the trio unit, the position of the person in between is fixed. Then, there are 2! ways to arrange the trio.
This gives a total of 6 * 6! * 2! arrangements.

Question 9: “Photographs”

8 people line up for a picture - how many ways to have <2 people between you and your friend?

Case 1, there are 0 people in between you and your friend => 7! * 2 arrangements

Case 2, there is 1 person in between you and your friend => 6 * 6! * 2! Arrangements

By the sum rule, there are 7! * 2 + 6 * 6! * 2 possible line-ups.

Questions?