

Section 1: Counting

Review of Main Concepts

- **Sum rule.** If you can choose from EITHER one of n options, OR one of m options with NO overlap with the previous n , then the number of possible outcomes of the experiment is _____.
- **Product rule.** In a sequential process with m steps, if there are n_1 choices for the 1st step, n_2 choices for the 2nd step (given the first choice), ..., and n_m choices for the m th step (given the previous choices), then the total number of outcomes is _____.
- **Permutations.** The number of ways to order n distinct elements is _____.
- **k -permutations.** The number of ways to choose a sequence of k distinct elements from a set of n elements is _____.
- **Subsets.** The number of ways to choose a k -element subset of a set of n elements is _____.
- **Set difference.** Is it always true that $|A \setminus B| = |A| - |B|$?
- **Complementary counting.** If asked to find the number of ways to do X, you can: find the _____ number of ways and then subtract the number of ways to _____.
- **Inclusion-exclusion.** $|A \cup B| = |A| + |B| - |A \cap B|$.
- **Inclusion-exclusion.** $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
- **Multinomial coefficients.** Suppose there are n objects, but only k are distinct, with $k \leq n$. (For example, “godoggy” has $n = 7$ objects (characters) but only $k = 4$ are distinct: (g, o, d, y)). Let n_i be the number of times object i appears, for $i \in \{1, 2, \dots, k\}$. (For example, $(3, 2, 1, 1)$, continuing the “godoggy” example.) The number of distinct ways to arrange the n objects is: _____.

1. Prerequisite Review: Sets

(a) For each one of the following sets, give its **cardinality**, i.e., indicate how many elements it contains:

- $A = \emptyset$
- $B = \{\emptyset\}$
- $C = \{\{\emptyset\}\}$
- $D = \{\emptyset, \{\emptyset\}\}$

(b) Let $S = \{a, b, c\}$ and $T = \{c, d\}$. Compute:

- $S \cup T$
- $S \cap T$
- $S \setminus T$
- $2^{S \setminus T}$
- $S \times T$

2. Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if ...

- (a) ...all couples are to get adjacent seats?
- (b) ...anyone can sit anywhere, except that one couple insists on *not* sitting in adjacent seats?

3. Weird Card Game

In how many ways can a pack of fifty-two cards be dealt to thirteen players, four to each, so that every player has one card of each suit?

4. HBCDEFGA

How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

5. Escape the Professor

There are 6 security professors and 7 theory professors taking part in an escape room. If 4 security professors and 4 theory professors are chosen and paired off, how many pairings are possible?

6. Birthday Cake

A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

7. Full Class

There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

8. Paired Finals

Suppose you are to take a CSE 312 final in pairs. There are 100 students in the class and 8 TAs, so 8 lucky students will get to pair up with a TA. Each TA must take the exam with some student, but two TAs cannot take the exam together. How many ways can they pair up?

9. Photographs

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

10. Rabbits!

Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

11. Extended Family Portrait

A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?

12. Lizards and Snakes!

Loudon has three pet lizards, Rango, a gecko named Gordon, and a goanna named Joanna, as well as two small pet snakes, Kaa and Basilisk, but only 4 terrariums to put them in. In how many different ways can he put his 5 pets in these 4 terrariums so that no terrarium has both a snake and a lizard?

13. Multinomial Coefficients

How many ways can we arrange the letters in 'TEDDYBEAR'?

14. Divide Me

How many numbers in $\{1, \dots, 360\}$ are divisible by:

- (a) 4, 6, and 9?
- (b) 4, 6, or 9?
- (c) Neither 4, 6, nor 9?