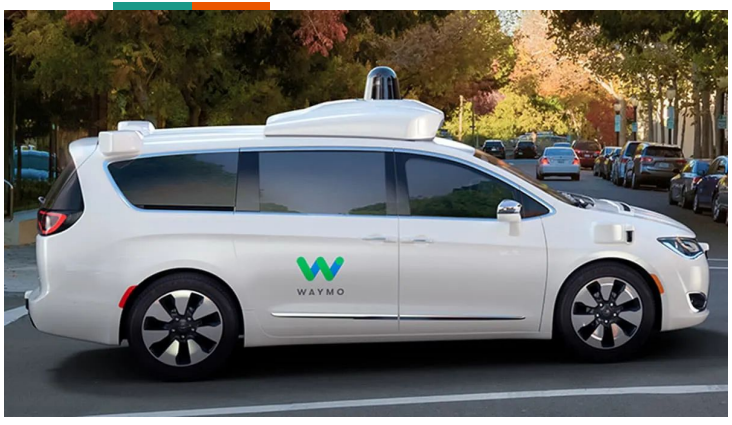
CSE 312 Spam Filter Intro

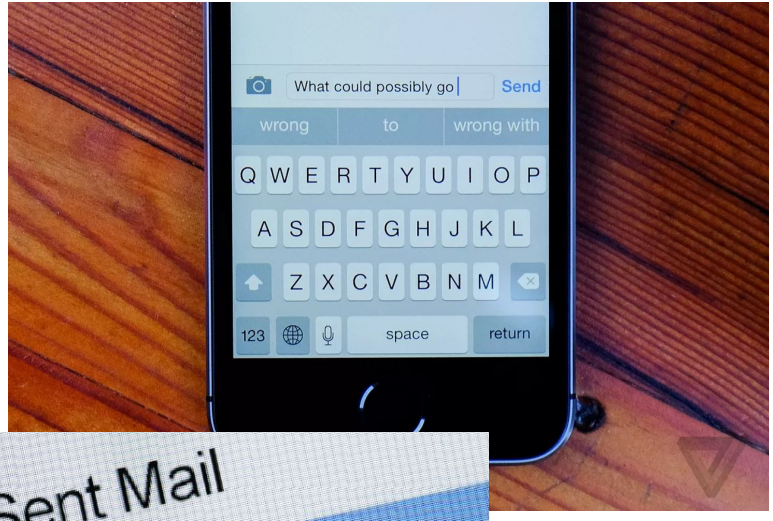
The Naive Bayes Classifier

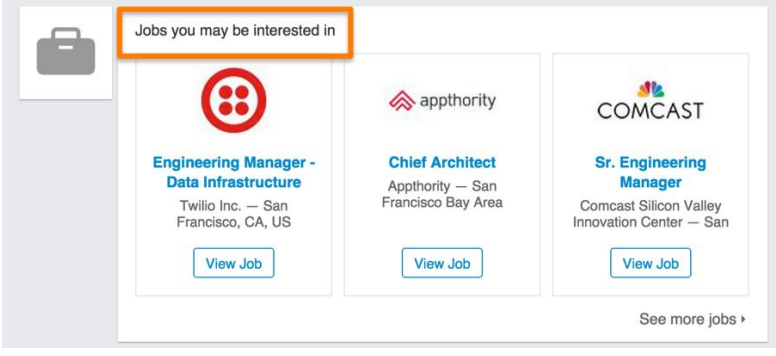
Agenda

- What is Machine Learning?
- Featurizing Emails
- Naive Bayes

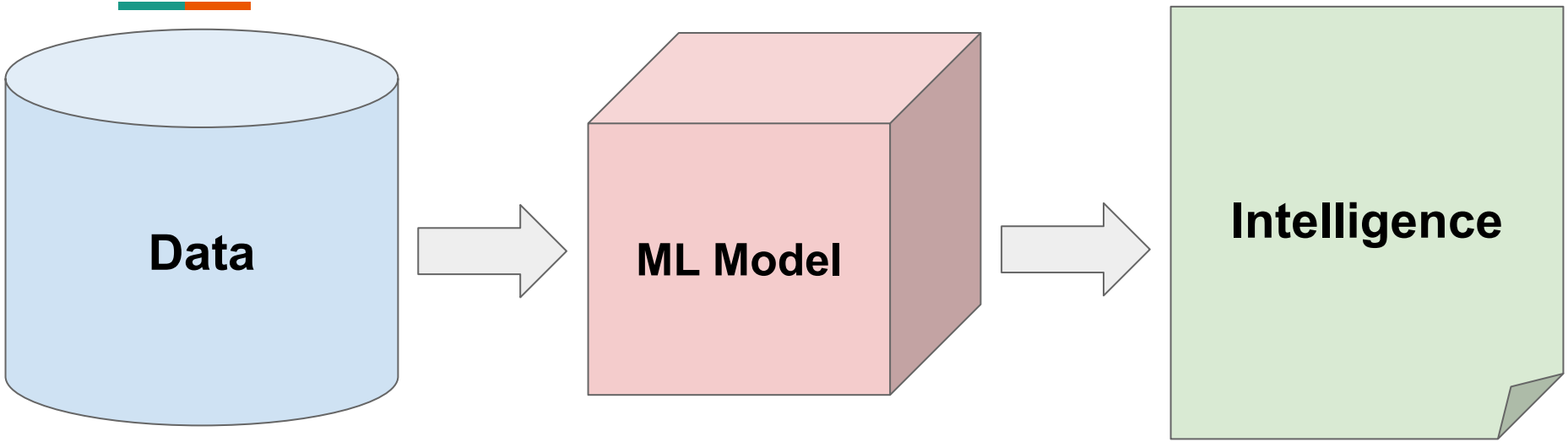
Machine Learning in the Real World







ML Pipeline



From **Wikipedia**: “Machine learning is the study of computer algorithms that improve automatically through experience.”

You are a machine!

Number	Shape	“Label”
3		12
5		15
-2		-8
7		21
-4		???

Given examples with correct “labels”, make predictions!

You are a machine!

Number	Shape	“Label”
3		12
5		15
-2		-8
7		21
-4		-16

Given examples with correct “labels”, make predictions!

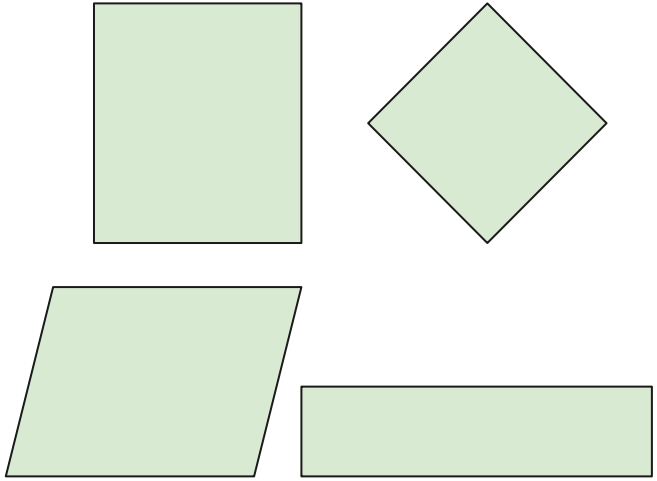
Regression: Idea

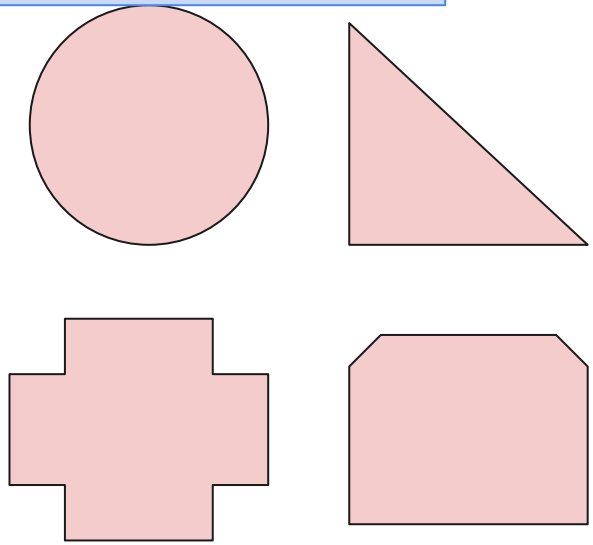
$ 340,135

$801,353

??????

Classification: Idea

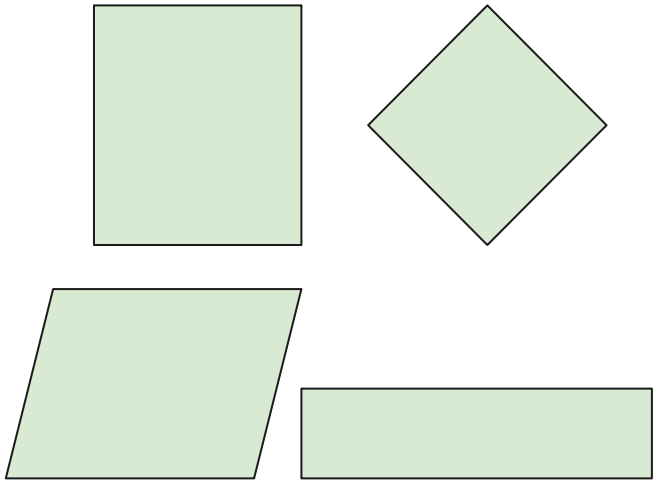


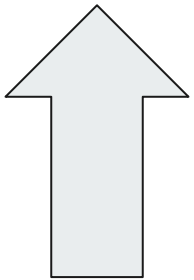


“Green” class

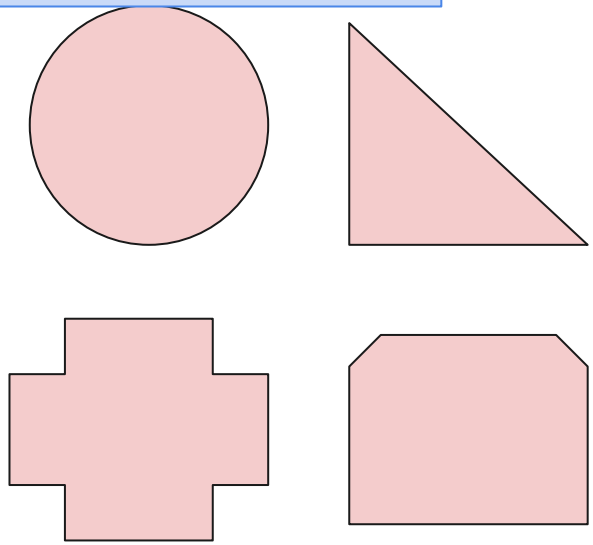
“Red” class

Classification: Idea





Is this new shape supposed to be “green” or “red”?

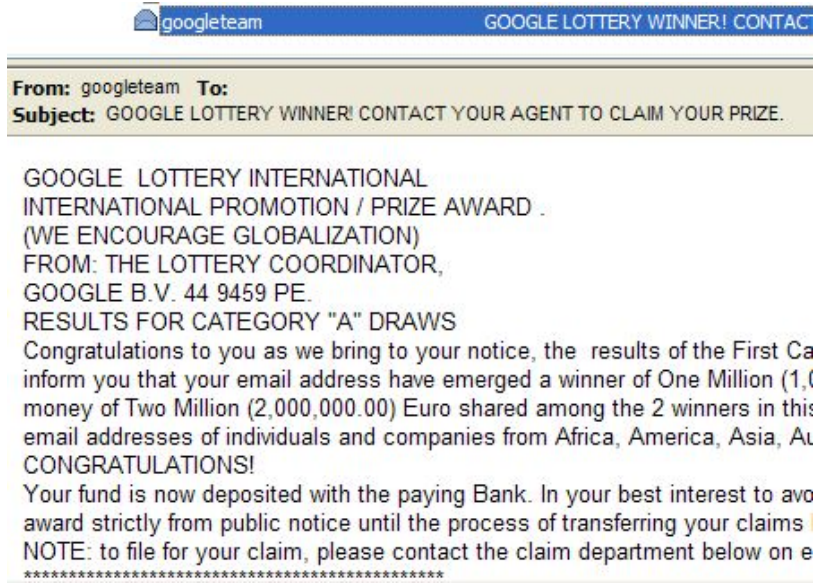


“Green” class

“Red” class

Spam Filter

- In real life, you may have seen a lot of spam emails like this.
- Building a good spam filter helps protect users from potential scams, unnecessary advertising, or malware links.



Evaluating Performance

Training Set

Email	Label
Buy Crypto!	Spam
You good?	Ham
Crypto help you.	Spam
Good Crypto help.	Spam
My Crypto wallet.	Ham

Test Set

Email	Label
You buy Crypto!	Spam
You need Crypto sir.	Spam
I hope you are financially healthy.	Ham
...	...
...	...

We “**train**” our spam filter on the training set, and **evaluate** performance using a test set (data that is unseen by the spam filter initially). This gives an unbiased estimate of performance.

Spam Filter Task

Training Set

Email	Label
Buy crypto!	Spam
You good?	Ham
Crypto help you.	Spam
Good Crypto help.	Spam
My Crypto wallet.	Ham

Predict whether this email is spam or ham:

You buy Crypto!

Emails as word collections

Email	Set of Words in the Email
SUBJECT: Top Secret Business Venture Dear Sir. First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret…	{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virtue, of, its, nature, as, being, utterly, confidencial, and}

For simplicity, we will

- Ignore Duplicate Words
- Ignore Punctuation
- Ignore Casing

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Hello hello hello there.	{hello, there}

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Hello hello hello there.	{hello, there}
You buy Crypto!	{you, buy, crypto}

For simplicity, we will

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Problem with this model

Consider for example the following two emails:

“!!!Lunch free for You!!!!!”

Spam

“You free for lunch?”

Ham

One shortfalling of our model is that it will make the same prediction for these since they have the same set of words!

Our approach

- Compute and Compare:

$$\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"}) \qquad \mathbb{P}(\text{ham} \mid \text{"You buy Crypto!"})$$

Then predict whichever is larger! Can we get away with just computing one of them?

Our approach

Compute and Compare:

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Then predict whichever is larger! Can we get away with just computing one of them?

Equivalently, note that these add to 1, so we can just compute $\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"})$

and if it is greater than 0.5, then we predict **spam**.

Otherwise, we predict **ham**.

Note: We resolve the tie in favor of **ham**.

Naive Bayes Classifier - The bayes part

Bayes Theorem:

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)}$$

Apply it to our example:

$$\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"}) = \frac{\mathbb{P}(\text{"You buy Crypto!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Crypto!"})}$$

Naive Bayes Classifier - What we Calculate

$$\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"}) = \frac{\mathbb{P}(\text{"You buy Crypto!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Crypto!"})}$$

Naive Bayes Classifier - What we Calculate

$$\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"}) = \frac{\mathbb{P}(\text{"You buy Crypto!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Crypto!"})}$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{ham})\mathbb{P}(\text{ham})}$$

Law of Total Probability (LTP): If events $E_1, \ldots, E_n$ partition Ω, then for any event F:

$$\mathbb{P}(F) = \sum_{i=1}^{n} \mathbb{P}(F \cap E_n) = \sum_{i=1}^{n} \mathbb{P}(F \mid E_i)\,\mathbb{P}(E_i)$$

Naive Bayes Classifier - What we Calculate

$$\mathbb{P}(\text{spam} \mid \text{"You buy Crypto!"}) = \frac{\mathbb{P}(\text{"You buy Crypto!"} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"You buy Crypto!"})}$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{ham})\mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}} \quad \mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

Naive Bayes Classifier - The naive part

How could we determine the value of this probability?

$$\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})$$

Naive Bayes Classifier - The naive part

It is somewhat unlikely that we have the email “You buy Crypto!” in our training data. (In this case we don’t!)

Naive Bayes Classifier - The naive part

It is somewhat unlikely that we have the email “You buy Crypto!” in our training data. (In this case we don’t!)

We **naively assume that words are conditionally independent from each other, given the label (In reality, they aren’t):** $\mathbb{P}(\boldsymbol{A}, \boldsymbol{B}|\boldsymbol{C}) = \mathbb{P}(\boldsymbol{A}|\boldsymbol{C})\mathbb{P}(\boldsymbol{B}|\boldsymbol{C})$

Conditional Independence

- In normal independence: we say two events A and B are independent of each other

$$\mathbb{P}(A, B) = \mathbb{P}(A)\,\mathbb{P}(B)$$

- In conditional independence, we say two events A and B are independent of each other *given* another event C

$$\mathbb{P}(A, B \mid C) = \mathbb{P}(A \mid C)\,\mathbb{P}(B \mid C)$$

  - What does this mean?
    - If we know whether an event C occurs or not, our knowledge about A tells us nothing about B (and vice versa)

Naive Bayes Classifier - The naive part

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$$\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\} \mid \text{spam})$$
$$\approx \mathbb{P}(\text{"you"} \mid \text{spam})\mathbb{P}(\text{"buy"} \mid \text{spam})\mathbb{P}(\text{"crypto"} \mid \text{spam})$$

Naive Bayes Classifier - The naive part

It is somewhat unlikely that we have the email ”You buy Crypto!” in our training data. (In this case we don’t!)

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$$\approx \mathbb{P}(\text{"you"} \mid \text{spam})\mathbb{P}(\text{"buy"} \mid \text{spam})\mathbb{P}(\text{"crypto"} \mid \text{spam})$$

Then we estimate that

Why is this Naive?

Words are not independent of each other!

Example

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"}) = \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

Email	Label
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$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

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$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"}|\text{spam}) = \frac{1}{3} \qquad \mathbb{P}(\text{"you"}|\text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"}|\text{spam}) = \frac{1}{3} \qquad \mathbb{P}(\text{"buy"}|\text{ham}) = 0$$

$$\mathbb{P}(\text{"crypto"}|\text{spam}) = 1 \qquad \mathbb{P}(\text{"crypto"}|\text{ham}) = \frac{1}{2}$$

Example

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

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$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

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$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{[illegible]})}$$

= 1

Email	Label
Buy Crypto!	Spam
You good?	Ham
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$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

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= 1 (Marked as spam since no ham email contained “buy”)

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What happens if we got a 0?

P(ham | “You buy Crypto!”) = 0 since P(“buy”| ham) = 0, since no ham email in our training data contained the word **‘buy’**.

But does that mean we will never encounter a ham email with word **‘buy’**?

Laplace smoothing

Pretend in spam emails (training set):

- We saw one extra spam email **with** word w_i
- We saw one extra spam email **without** word w_i

Laplace smoothing

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- We saw one extra spam email **with** word w_i
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$$\mathbb{P}(w_i \mid \text{spam}) = \frac{\left|\text{total spam emails (training set) containing } w_i\right| + 1}{\left|\text{total spam emails (training set)}\right| + 2}$$

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Same for ham emails:

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Laplace smoothing

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$$\mathbb{P}(\text{“buy”} \mid \text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

Example

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

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My Crypto wallet.	Ham

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$$\mathbb{P}(\text{"you"}|\text{spam}) = \frac{1}{3} \qquad \mathbb{P}(\text{"you"}|\text{ham}) = \frac{1}{2}$$

$$\mathbb{P}(\text{"buy"}|\text{spam}) = \frac{1}{3} \qquad \mathbb{P}(\text{"buy"}|\text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"crypto"}|\text{spam}) = 1 \qquad \mathbb{P}(\text{"crypto"}|\text{ham}) = \frac{1}{2}$$

Example

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

Email	Label
Buy Crypto!	Spam
You good?	Ham
Crypto help you.	Spam
Good Crypto help.	Spam
My Crypto wallet.	Ham

$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

$$\mathbb{P}(\text{"you"}|\text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{"you"}|\text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

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$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

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$$\mathbb{P}(\text{"buy"}|\text{spam}) = \frac{1+1}{3+2} = \frac{2}{5} \qquad \mathbb{P}(\text{"buy"}|\text{ham}) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"crypto"}|\text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"crypto"}|\text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Example

Email	Label
Buy Crypto!	Spam
You good?	Ham
Crypto help you.	Spam
Good Crypto help.	Spam
My Crypto wallet.	Ham

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"}) = \frac{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"},\text{"buy"},\text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"},\text{"buy"},\text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5}}{\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{4}{5}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{2}{5}} \approx 0.7544$$

$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

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$$\mathbb{P}(\text{"crypto"}|\text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"crypto"}|\text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Example

$$\mathbb{P}(\text{spam}|\text{"You buy Crypto"})$$

$$= \frac{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\{\text{"you"}, \text{"buy"}, \text{"crypto"}\}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam})}{\mathbb{P}(\text{"you"}|\text{spam})\mathbb{P}(\text{"buy"}|\text{spam})\mathbb{P}(\text{"crypto"}|\text{spam})\mathbb{P}(\text{spam}) + \mathbb{P}(\text{"you"}|\text{ham})\mathbb{P}(\text{"buy"}|\text{ham})\mathbb{P}(\text{"crypto"}|\text{ham})\mathbb{P}(\text{ham})}$$

$$= \frac{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5}}{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{5}} \approx 0.7544$$

Email	Label
Buy Crypto!	Spam
You good?	Ham
Crypto help you.	Spam
Good Crypto help.	Spam
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$$\mathbb{P}(\text{spam}) = \frac{3}{5} \qquad \mathbb{P}(\text{ham}) = \frac{2}{5}$$

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$$\mathbb{P}(\text{"crypto"}|\text{spam}) = \frac{3+1}{3+2} = \frac{4}{5} \qquad \mathbb{P}(\text{"crypto"}|\text{ham}) = \frac{1+1}{2+2} = \frac{1}{2}$$

Underflow Prevention

- Multiplication of many probabilities, each of which will be between 0 and 1, can result in floating-point underflow. The product will be too small and will result in arithmetic underflow.

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Underflow Prevention

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- Reminder: Log property:

$$\log(xy) = \log(x) + \log(y)$$

- Summing logs of probabilities is better than multiplying probabilities

$$\log\left(\prod_{i=1}^{n} p_i\right) = \log(p_1 p_2 \ldots p_n) = \log(p_1) + \log(p_2) + \ldots + \log(p_n)$$
$$= \sum_{i=1}^{n} \log(p_i)$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\})$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\ \mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\ \mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\})$$
$$\iff \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{ham})\mathbb{P}(\text{ham})$$

Denominators are equal and cancel when comparing

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\,\mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\,\mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\,\mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\,\mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{spam})\,\mathbb{P}(\text{spam}) + \mathbb{P}(\{w_1,w_2,\ldots,w_n\} \mid \text{ham})\,\mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\begin{aligned}
&\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \\
\iff\ &\mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{ham})\mathbb{P}(\text{ham}) \\
\iff\ &\mathbb{P}(w_1 \mid \text{spam})\mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham})\mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham})\mathbb{P}(\text{ham})
\end{aligned}$$

Applying underflow prevention

$$\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ spam})\,\mathbb{P}(\text{spam})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ spam})\,\mathbb{P}(\text{spam})+\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ ham})\,\mathbb{P}(\text{ham})}$$

$$\mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \approx \frac{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ ham})\,\mathbb{P}(\text{ham})}{\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ spam})\,\mathbb{P}(\text{spam})+\mathbb{P}(\{w_1,w_2,\ldots,w_n\}|\text{ ham})\,\mathbb{P}(\text{ham})}$$

We will output **spam** iff:

$$\begin{aligned}
&\mathbb{P}(\text{spam} \mid \{w_1, w_2, \ldots, w_n\}) > \mathbb{P}(\text{ham} \mid \{w_1, w_2, \ldots, w_n\}) \\
\iff\ &\mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(\{w_1, w_2, \ldots, w_n\} \mid \text{ham})\mathbb{P}(\text{ham}) \\
\iff\ &\mathbb{P}(w_1 \mid \text{spam})\mathbb{P}(w_2 \mid \text{spam}) \cdots \mathbb{P}(w_n \mid \text{spam})\mathbb{P}(\text{spam}) > \mathbb{P}(w_1 \mid \text{ham})\mathbb{P}(w_2 \mid \text{ham}) \cdots \mathbb{P}(w_n \mid \text{ham})\mathbb{P}(\text{ham})
\end{aligned}$$

Taking the log of two sides:

$$\iff \log(\mathbb{P}(\text{spam}))+\sum_{i=1}^{n}\log(\mathbb{P}(w_i \mid \text{spam}) > \log(\mathbb{P}(\text{ham}))+\sum_{i=1}^{n}\log(\mathbb{P}(w_i \mid \text{ham})$$

Summary: Naive Bayes Algorithm steps

1. **TRAINING**

   1.1. Compute the proportion of emails in the **training set** that is spam or ham:

$$\mathbb{P}(\text{spam}) = \frac{\text{total spam emails (in training set)}}{\text{total emails (in training set)}} \qquad \mathbb{P}(\text{ham}) = \frac{\text{total ham emails (in training set)}}{\text{total emails (in training set)}}$$

   1.2. Iterate over the **training set**, for each unique word **x**, count:

   - How many **spam emails** in the training set contain **x**
   - How many **ham emails** in the training set contain **x**

Summary: Naive Bayes Algorithm steps

2. **TESTING**

Iterate over the **test set**, for each unlabelled email **D**:

- Create a set **S** of **n** unique words appearing in **D**: $\{w_1, w_2, \ldots, w_n\}$
- For each word w_i in set **S**, calculate:

$$\mathbb{P}(w_i|spam) = \frac{|\text{total spam emails (training set) containing } w_i| + 1}{\text{total spam emails (training set)} + 2} \qquad \mathbb{P}(w_i \mid \text{ham}) = \frac{|\text{total ham emails (training set) containing } w_i| + 1}{|\text{total ham emails (training set)}| + 2}$$

  - Note: If word w_i doesn't appear in the training set, we still calculate the above probabilities, with:

$$|\text{total spam emails (training set) containing } w_i| = |\text{total ham emails (training set) containing } w_i| = 0$$

- if $\log(\mathbb{P}(\text{spam})) + \sum_{i=1}^{n} \log(\mathbb{P}(w_i \mid \text{spam})) > \log(\mathbb{P}(\text{ham})) + \sum_{i=1}^{n} \log(\mathbb{P}(w_i \mid \text{ham}))$

Predict email D as **spam**
Otherwise, predict email D as **ham**

Questions?
Comments?
Concerns?