Conditional Probability

CSE 312 Su23
Lecture 5

Where are we?

Last class, we started calculating probabilities.

Today, we look at what happens when we introduce conditioning.

Conditional Probabilities

Conditioning

You roll a fair red die and a fair blue die (without letting the dice affect each other).

But they fell off the table and you can't see the results.

What is the probability the red die shows a 5?

Conditioning

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But they fell off the table and you can't see the results.

What is the probability the red die shows a 5? $1/6$.

Conditioning

You roll a fair red die and a fair blue die (without letting the dice affect each other).

But they fell off the table and you can't see the results.

I can see the results – I tell you the sum of the two dice is 4.

What's the probability that the red die shows a 5, **conditioned** on knowing the sum is 4?

Conditioning

You roll a fair red die and a fair blue die (without letting the dice affect each other).

But they fell off the table and you can't see the results.

I can see the results – I tell you the sum of the two dice is 4.

What's the probability that the red die shows a 5, **conditioned** on knowing the sum is 4?

It's 0.

Without the conditioning it was $1/6$.

Conditioning

When I told you "the sum of the dice is 4" we restricted the sample space.

The only remaining outcomes are $\{(1,3),(2,2),(3,1)\}$ out of $\{1,2,3,4,5,6\}\times\{1,2,3,4,5,6\}$.

Outside the (restricted) sample space, the probability is going to become 0. What about the probabilities inside?

Conditional Probability

Conditional Probability

For an event B, with $\mathbb{P}(B) > 0$,
the "Probability of A conditioned on B" is

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Just like with the formal definition of probability, this is pretty abstract.
It does accurately reflect what happens in the real world.

If $\mathbb{P}(B) = 0$, we can't condition on it (it can't happen! There's no point in defining probabilities where we know B has happened) – $\mathbb{P}(A|B)$ is **undefined** when $\mathbb{P}(B) = 0$.

Conditioning…

Let A be “the red die is 5”
Let B be “the sum is 4”
Let C be “the blue die is 3”

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
D1=1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1.6)
D1=2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
D1=3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
D1=4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
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Conditioning…

Let A be "the red die is 5"
Let B be "the sum is 4"
Let C be "the blue die is 3"

$\mathbb{P}(A|B)$

$\mathbb{P}(A \cap B) = \mathbb{P}(\emptyset) = 0$
$\mathbb{P}(B) = 3/36$

$$P(A|B) = \frac{0}{3/36}$$

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
D1=1	(1,1)	(1,2)	$\boxed{(1,3)}$	(1,4)	(1,5)	(1.6)
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Conditioning…

Let A be "the red die is 5"
Let B be "the sum is 4"
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$\mathbb{P}(A|C)$

$\mathbb{P}(A \cap C) = 1/36$
$\mathbb{P}(C) = 6/36$

$$P(A|C) = \frac{1/36}{6/36}$$

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
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Conditioning Practice

Sum 7 conditioned on red die 6?

$$\mathbb{P}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbb{P}(\boldsymbol{B})}$$

	D2=1	D2=2	D2=3	D2=4	D2=5	D2=6
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Conditioning Practice

Sum 7 conditioned on red die 6 1/6

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Direction Matters

$\mathbb{P}(A|B)$ and $\mathbb{P}(B|A)$ are different quantities.

$\mathbb{P}($"I'm doing homework" | "I have homework"$)$ is pretty small [Don't be like me]

$\mathbb{P}($"I have homework" | "I'm doing homework"$)$ is 1 (or close to it); there's no other situation that would make me do homework ☺

It's a lot like implications – order can matter a lot!

(but there are some A, B where the conditioning doesn't make a difference)

Wonka Bars

Willy Wonka has placed golden tickets on 0.1% (1/1000) of his Wonka Bars.

You want to get a golden ticket. You could buy a 1000-or-so of the bars until you find one, but that’s expensive…you’ve got a better idea!

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You want to get a golden ticket. You could buy a 1000-or-so of the bars until you find one, but that's expensive…you've got a better idea!

You have a test – a very precise scale you've bought.

- If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.
- If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

If you pick up a bar and it alerts, what is the probability you have a golden ticket?

Willy Wonka

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If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.

If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

If you pick up a bar and it alerts, what is the probability you have a golden ticket?

Which of these is closest to the right answer?

A. 0.1%
B. 10%
C. 50%
D. 90%
E. 99%
F. 99.9%

Conditioning

Let A be the event you get ALERTED

Let T be the event your bar has a ticket.

What conditional probabilities are each of these?

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

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Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

$\boxed{\mathbb{P}(T) = 0.001}$

If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.

$\boxed{\mathbb{P}(A|T) = 0.999}$

If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

$\boxed{\mathbb{P}(A|\bar{T}) = 0.01}$

If you pick up a bar and it alerts, what is the probability you have a golden ticket?

$\boxed{\mathbb{P}(T|A) = ?}$

Reversing the Conditioning

All of our information conditions on whether B happens or not – does your bar have a golden ticket or not?

$\boxed{\mathbb{P}(A|T) = 0.999}$ $\boxed{\mathbb{P}(A|\bar{T}) = 0.01}$

But we're interested in the "reverse" conditioning. We know the scale alerted us – we know the test is positive – but do we have a golden ticket?

We want: $\boxed{\mathbb{P}(T|A) = ?}$

But the givens give us information with **reversed** conditioning.

Is there a relation between the two?

Bayes Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Bayes Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

From earlier, we have:

$\mathbb{P}(T) = 0.001$ | $\mathbb{P}(A|T) = 0.999$ | $\mathbb{P}(A|\bar{T}) = 0.01$ | $\mathbb{P}(T|A) = ?$

Plugging in values,

$$\mathbb{P}(T|A) = \frac{\mathbb{P}(A|T)\mathbb{P}(T)}{P(A)} = \frac{0.999 \cdot 0.001}{\mathbb{P}(A)}$$

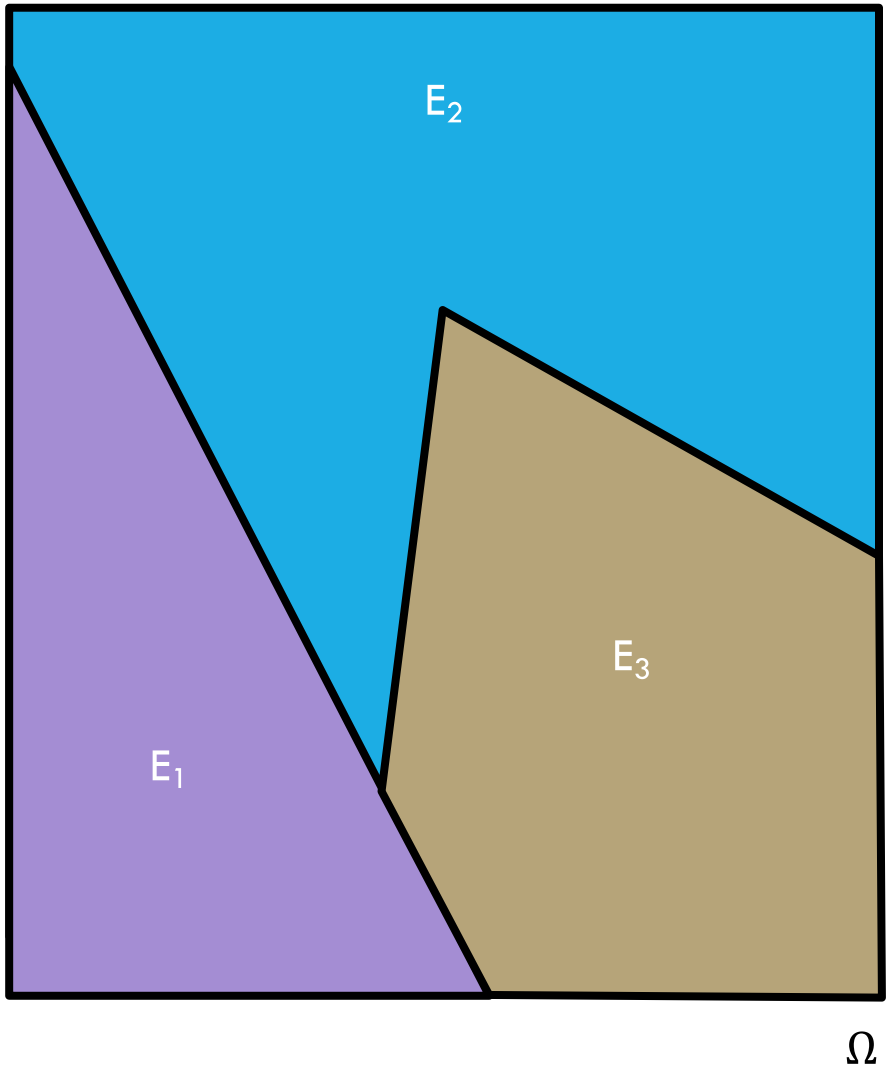
Filling In

What's $\mathbb{P}(A)$?

We need another rule, called "law of total probability"!

Let's take a small detour to understand this law before returning to this question ☺

Partitions



E_1, E_2 and E_3 **partition** the sample space.

What exactly does that mean though?

1. They take up the entire sample space, or "**exhaust**" the sample space.

2. They do not overlap, or they are (pairwise) "**mutually exclusive**".

Partitions, more formally

Partition

Non-empty events $E_1, E_2, \ldots, E_n$ **partition** the sample space Ω if they are both:

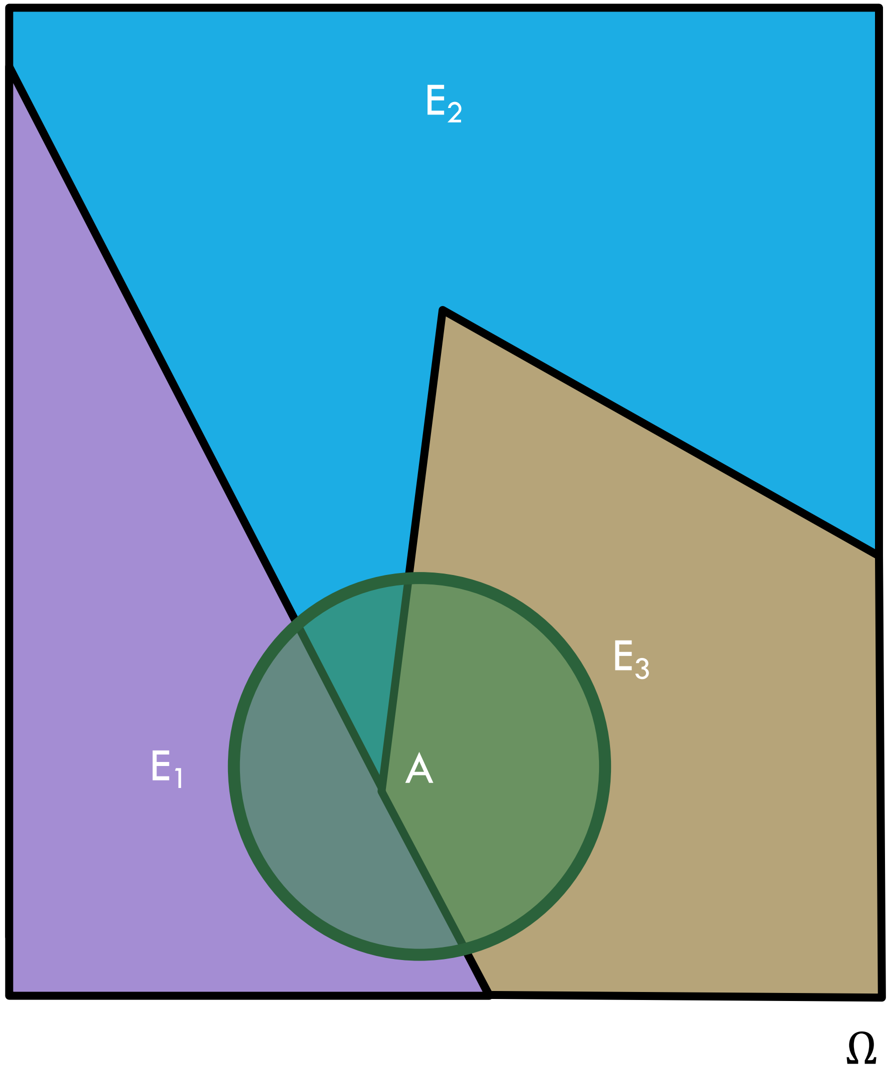
- **Exhaustive**

$$E_1 \cup E_2 \cup \cdots \cup E_n = \cup_{i=1}^{n} E_i = \Omega$$

- **Pairwise Mutually Exclusive**

$$\forall i \, \forall j \; i \neq j \,, E_i \cap E_j = \emptyset$$

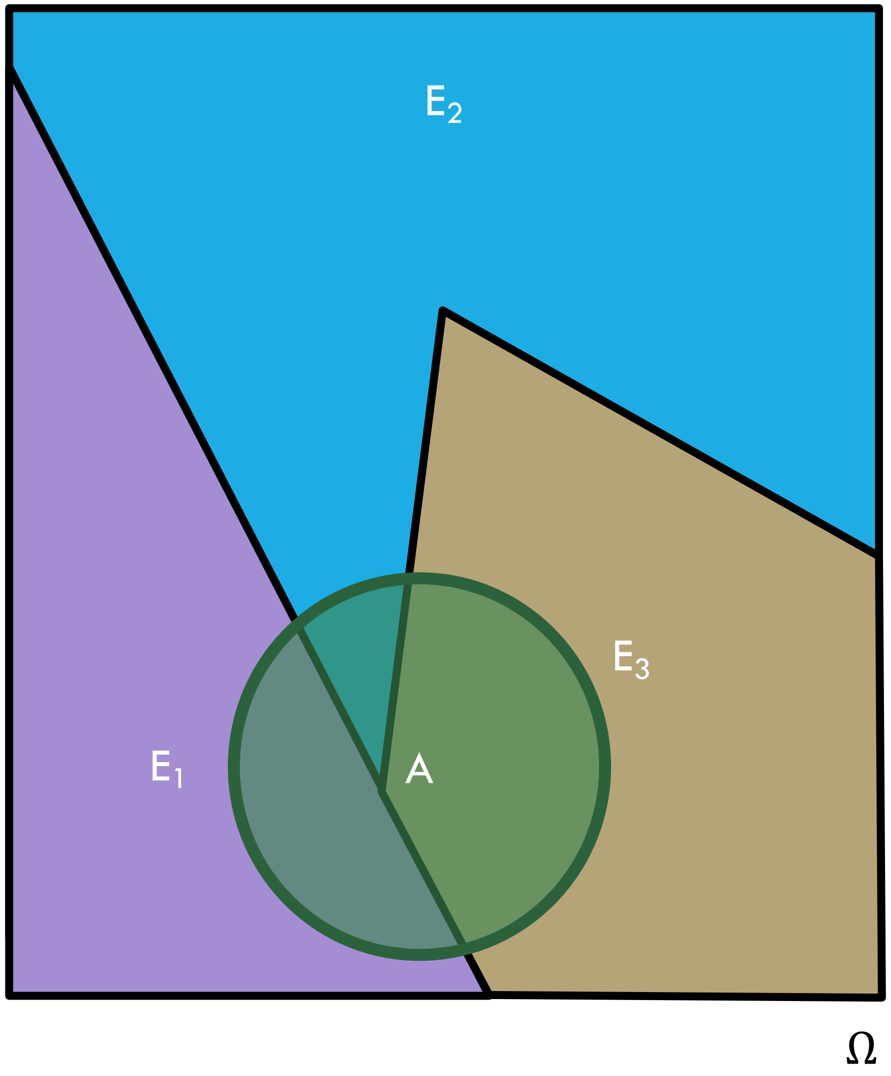
Law of Total Probability, Intuition



E_1, E_2 and E_3 partition Ω.

What can we say about event A?

Law of Total Probability, Intuition



E_1, E_2 and E_3 partition Ω.

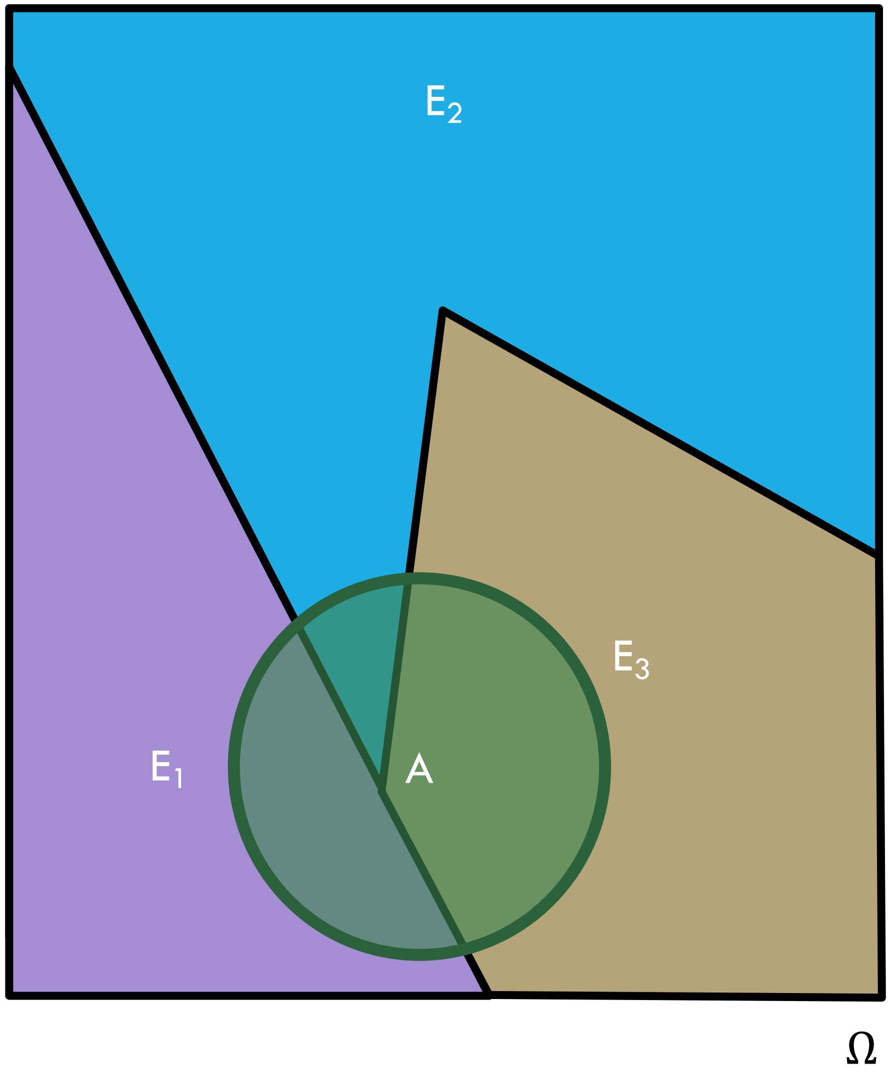
What can we say about event A?

$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3)$

So,

$\mathbb{P}(A) = \mathbb{P}(A \cap E_1) + \mathbb{P}(A \cap E_2) + \mathbb{P}(A \cap E_3)$

Law of Total Probability, Intuition



$$\mathbb{P}(A) = \mathbb{P}(A \cap E_1) + \mathbb{P}(A \cap E_2) + \mathbb{P}(A \cap E_3)$$

From the definition of conditional prob.,

$$\mathbb{P}(A \cap E_1) = P(A|E_1) \cdot P(E_1)$$

So, equivalently,

$$\mathbb{P}(A) = \mathbb{P}(A|E_1)\mathbb{P}(E_1) + \mathbb{P}(A|E_2)\mathbb{P}(E_2) + \mathbb{P}(A|E_3)\mathbb{P}(E_3)$$

Law of Total Probability

Law of Total Probability

Let $E_1, E_2, \ldots, E_n$ be a **partition** of Ω.

For any event A,

$$\mathbb{P}(A) = \sum_{i=1}^{n} \mathbb{P}(A \cap E_1) = \sum_{i=1}^{n} \mathbb{P}(A|E_1)\mathbb{P}(E_1)$$

Now that you’ve completely forgotten why we needed this…

Conditioning

Let A be the event you get ALERTED

Let B be the event your bar has a ticket.

What conditional probabilities are each of these?

Willy Wonka has placed golden tickets on 0.1% of his Wonka Bars.

$\boxed{\mathbb{P}(T) = 0.001}$

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$\boxed{\mathbb{P}(A|T) = 0.999}$

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$\boxed{\mathbb{P}(A|\bar{T}) = 0.01}$

If you pick up a bar and it alerts, what is the probability you have a golden ticket?

$\boxed{\mathbb{P}(T|A) = ?}$

Bayes Rule

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

From earlier, we have:

$\mathbb{P}(T) = 0.001$ | $\mathbb{P}(A|T) = 0.999$ | $\mathbb{P}(A|\bar{T}) = 0.01$ | $\mathbb{P}(T|A) = ?$

Plugging in values,

$$\mathbb{P}(T|A) = \frac{\mathbb{P}(A|T)\mathbb{P}(T)}{P(A)} = \frac{0.999 \cdot 0.001}{\mathbb{P}(A)}$$

Filling In

What's $\mathbb{P}(A)$?

We can partition the sample space into T and $\bar{T}$ (i.e., the event the bar has a ticket and the event it doesn't)

[Note that these events exhaust the sample space and are mutually exclusive!]

$$\mathbb{P}(A) = \mathbb{P}(A|T) \cdot \mathbb{P}(T) + \mathbb{P}(A|\bar{T}) \cdot P(\bar{T}) \text{ [LTP]}$$

$$= 0.999 \cdot .001 + .01 \cdot .999$$

$$= .010989$$

$\mathbb{P}(T) = 0.001$

$\mathbb{P}(A|T) = 0.999$

$\mathbb{P}(A|\bar{T}) = 0.01$

Bayes Rule

Plugging in values,

$$\mathbb{P}(T|A) = \frac{\mathbb{P}(A|T)\mathbb{P}(T)}{P(A)} = \frac{0.999 \cdot 0.001}{\mathbb{P}(A)} = \frac{0.999 \cdot 0.001}{0.999 \cdot 0.001 + 0.01 \cdot 0.999}$$

$$= \frac{1}{11} \cong 0.0909$$

Only about a 10% chance that the bar has the golden ticket!

Wait a minute…

Willy Wonka has placed golden tickets on **0.1%** of his Wonka Bars.
If the bar you weigh **does** have a golden ticket, the scale will alert you 99.9% of the time.
If the bar you weigh does not have a golden ticket, the scale will (falsely) alert you only 1% of the time.

That doesn't fit with many of our guesses. What's going on?

Instead of thinking about 1 bar, imagine we test 1000 bars.

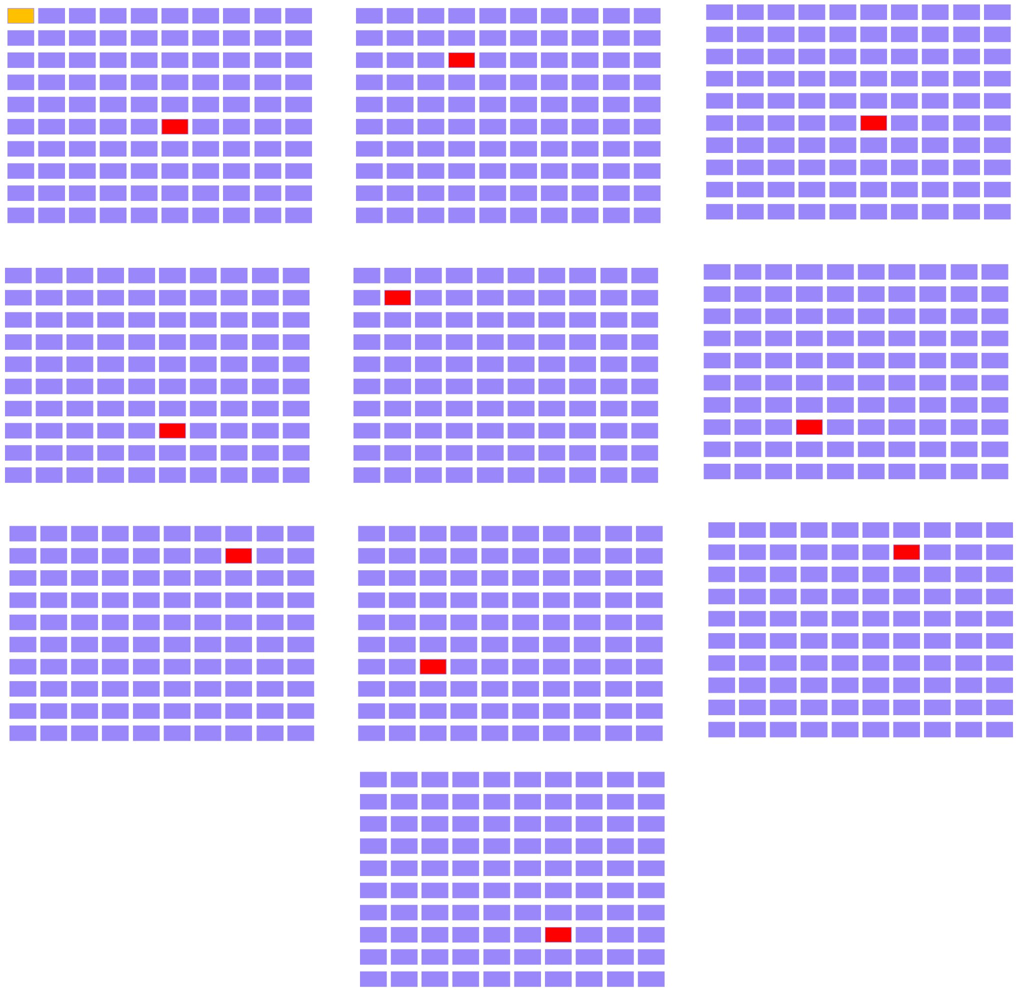
Of those, (**about**) 1 has a golden ticket, while the remaining 999 don't.

Assuming those numbers hold…

For the 1 with a ticket, let's say the scale alerts us (correctly).

For the remaining 999 bars, we get a false alert 1% of the time – or for say, 10 bars.

Visually



Gold bar is the one (true) golden ticket bar.
Purple bars don't have a ticket and tested negative.
Red bars don't have a ticket but tested positive.

The test is, in a sense, doing really well.
It's almost always right.

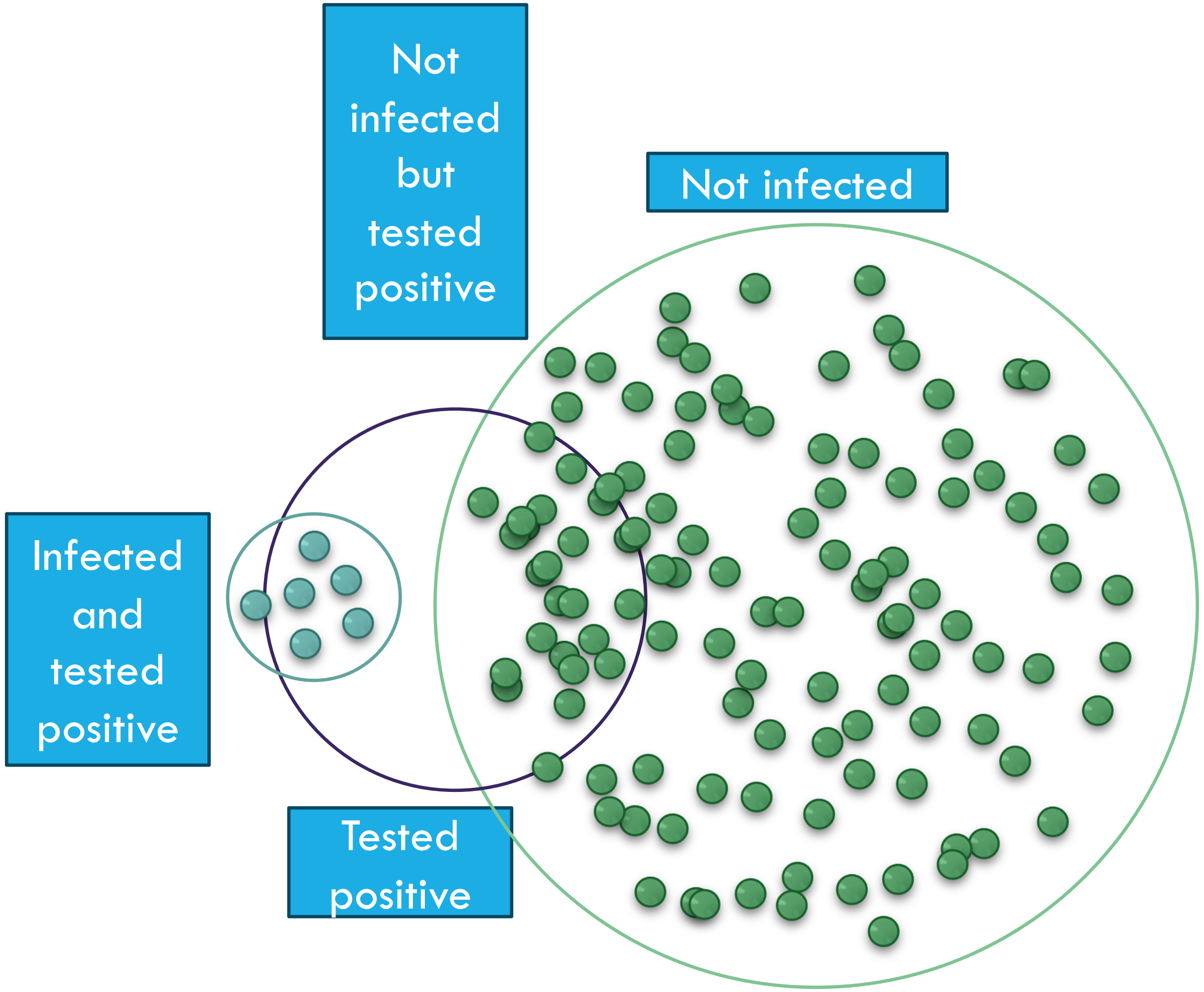
The problem is: it's also the case that the correct answer is almost always "no."

Base Rate Fallacy

We cannot look only at the **accuracy** of the test but must also look at the sampled population.

When the proportion of 'true positives' in a population is lower than the test's false positive rate, even tests that have a low false positive rate will give more false than true positives overall.

Why does this matter? Think about the contexts where we rely on these scientific tests, like medicine and law (esp. criminal).



Updating Your Intuition

🔥 Take 1: The test is **actually good** and has VASTLY increased our belief that there IS a golden ticket when you get a positive result.

If we told you "your job is to find a Wonka Bar with a golden ticket" without the test, you have 1/1000 chance, with the test, you have (about) a 1/11 chance. That's (almost) 100 times better!

This is actually a huge improvement!

Updating Your Intuition

🔥 Take 2: Humans are really bad at intuitively understanding very large or very small numbers.

When I hear “99% chance”, “99.9% chance”, “99.99% chance” they all go into my brain as “well that’s basically guaranteed” And then I forget how many 9’s there actually were.

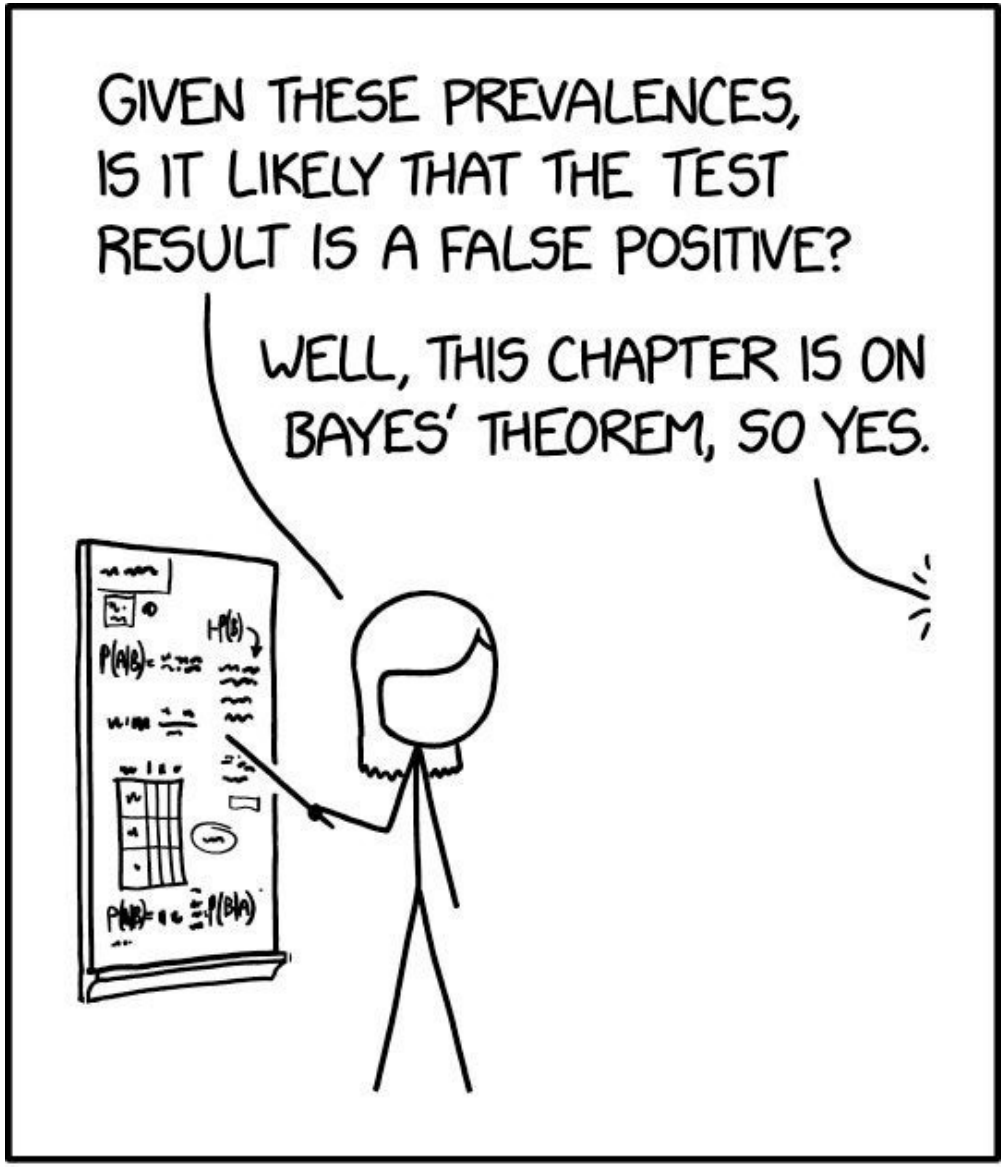
But the number of 9s matters because they end up “cancelling” with the “number of 9’s” in the population that’s truly negative.

Updating Your Intuition

🔥 Take 3: View tests as updating your beliefs, not as revealing the truth.

Bayes' Rule says that $\mathbb{P}(B|A)$ has a factor of $\mathbb{P}(B)$ in it. You have to translate "The test says there's a golden ticket" to "the test says you should increase your estimate of the chances that you have a golden ticket."

A test takes you from your "prior" beliefs of the probability to your "posterior" beliefs.



SOMETIMES, IF YOU UNDERSTAND BAYES' THEOREM WELL ENOUGH, YOU DON'T NEED IT.

https://xkcd.com/2545/

More Bayes Practice

A contrived example

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.

You will flip a fair coin; if it's heads, you'll draw a marble (uniformly) from your left pocket, if it's tails, you'll draw a marble (uniformly) from your right pocket.

Let B be you draw a blue marble. Let T be the coin is tails.

What is $\mathbb{P}(B|T)$? What is $\mathbb{P}(T|B)$?

Updated Sequential Processes

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
if it's heads, you'll draw a marble (uniformly) from your left pocket,
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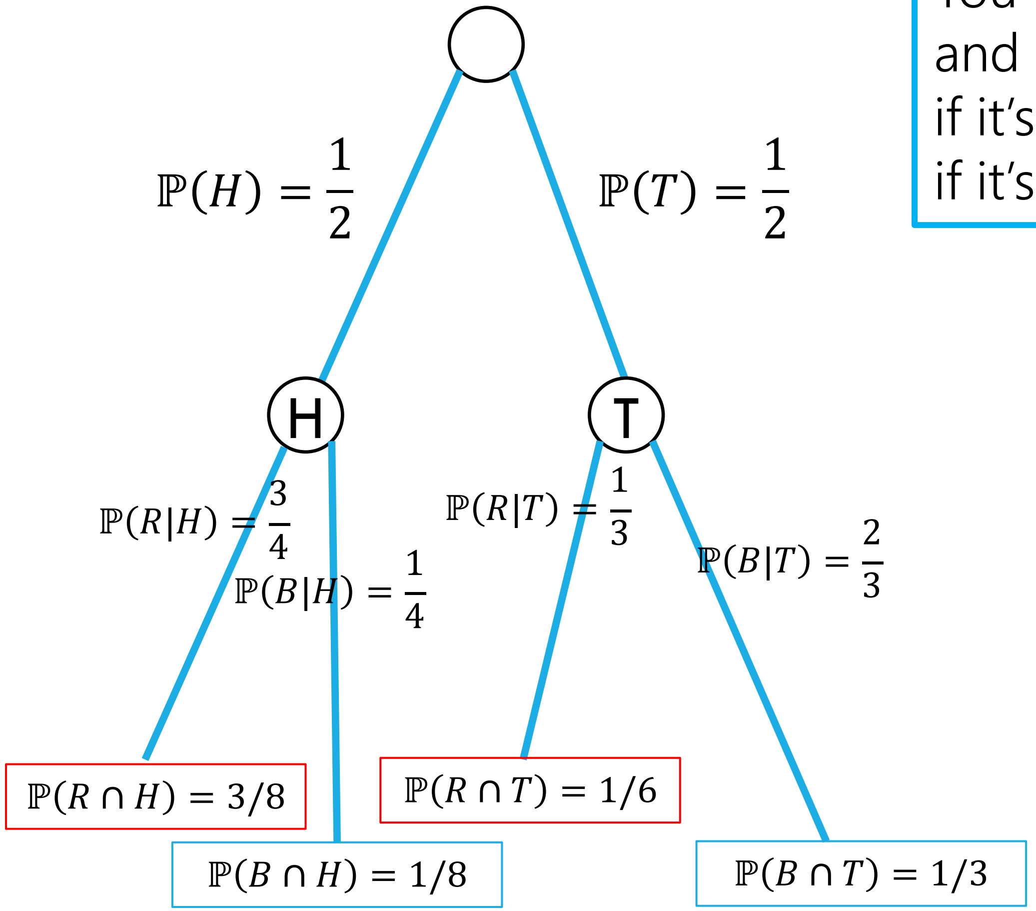

For sequential processes with probability, at each step multiply by
$\mathbb{P}(\text{next step } | \text{all} \cap \text{prior} \cap \text{steps})$

Updated Sequential Processes

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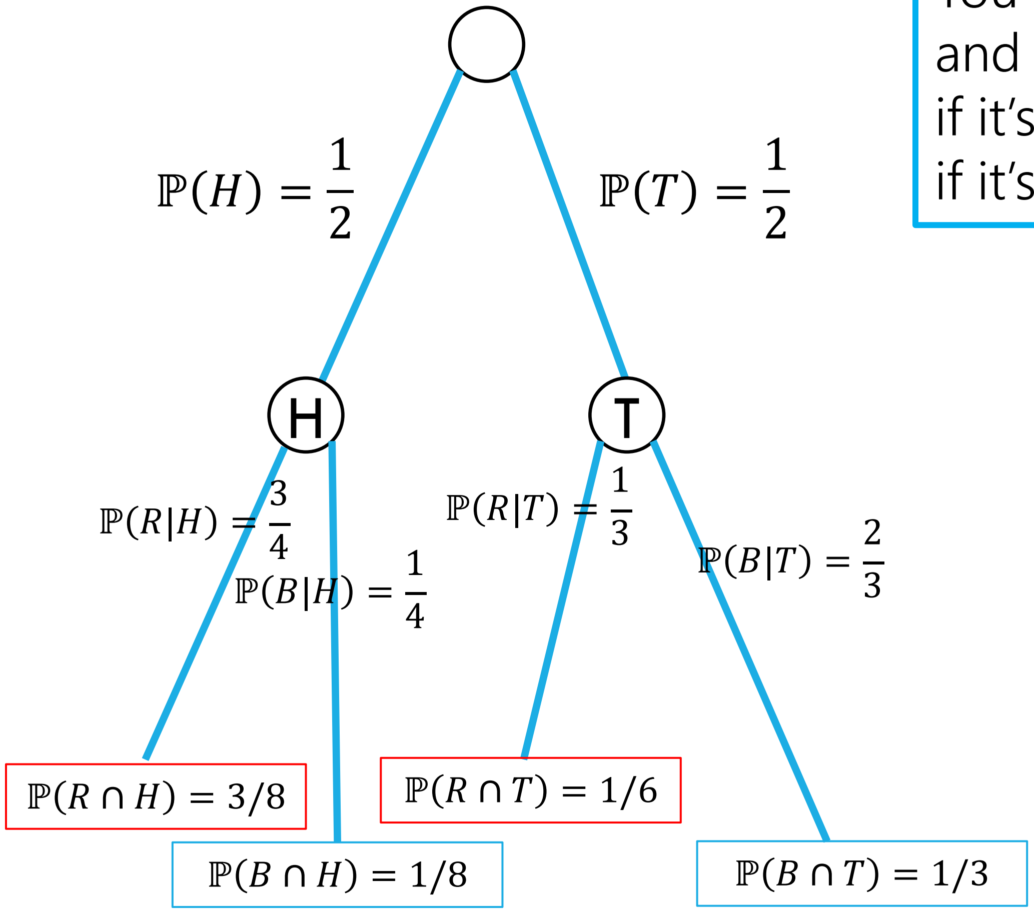

For sequential processes with probability, at each step multiply by
$\mathbb{P}(\text{next step } | \text{all} \cap \text{prior} \cap \text{steps})$

$$\mathbb{P}(B|T) = 2/3;\ \mathbb{P}(B) = \frac{1}{8} + \frac{1}{3} = \frac{11}{24}$$

Flipping the conditioning

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
if it's heads, you'll draw a marble (uniformly) from your left pocket,
if it's tails, you'll draw a marble (uniformly) from your right pocket.

What about $\mathbb{P}(T|B)$?

Pause, what's your intuition?

Is this probability

A. less than ½

B. equal to ½

C. greater than ½

Flipping the conditioning

What about $\mathbb{P}(T|B)$?

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
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Pause, what's your intuition?

Is this probability

A. less than ½

B. equal to ½

C. greater than ½

The right (tails) pocket is far more likely to produce a blue marble if picked than the left (heads) pocket is. Seems like $\mathbb{P}(T|B)$ should be greater than ½.

Flipping the conditioning

What about $\mathbb{P}(T|B)$?

You have three red marbles and one blue marble in your left pocket, and one red marble and two blue marbles in your right pocket.
if it's heads, you'll draw a marble (uniformly) from your left pocket,
if it's tails, you'll draw a marble (uniformly) from your right pocket.

Bayes' Rule says:

$$\mathbb{P}(T|B) = \frac{\mathbb{P}(B|T)\mathbb{P}(T)}{\mathbb{P}(B)}$$

$$= \frac{\frac{2}{3} \cdot \frac{1}{2}}{11/24} = 8/11$$

Takeaway

Bayes' Theorem is useful when we want to estimate some **unobservable** true state (our belief) using some **observable** evidence… in a variety of contexts

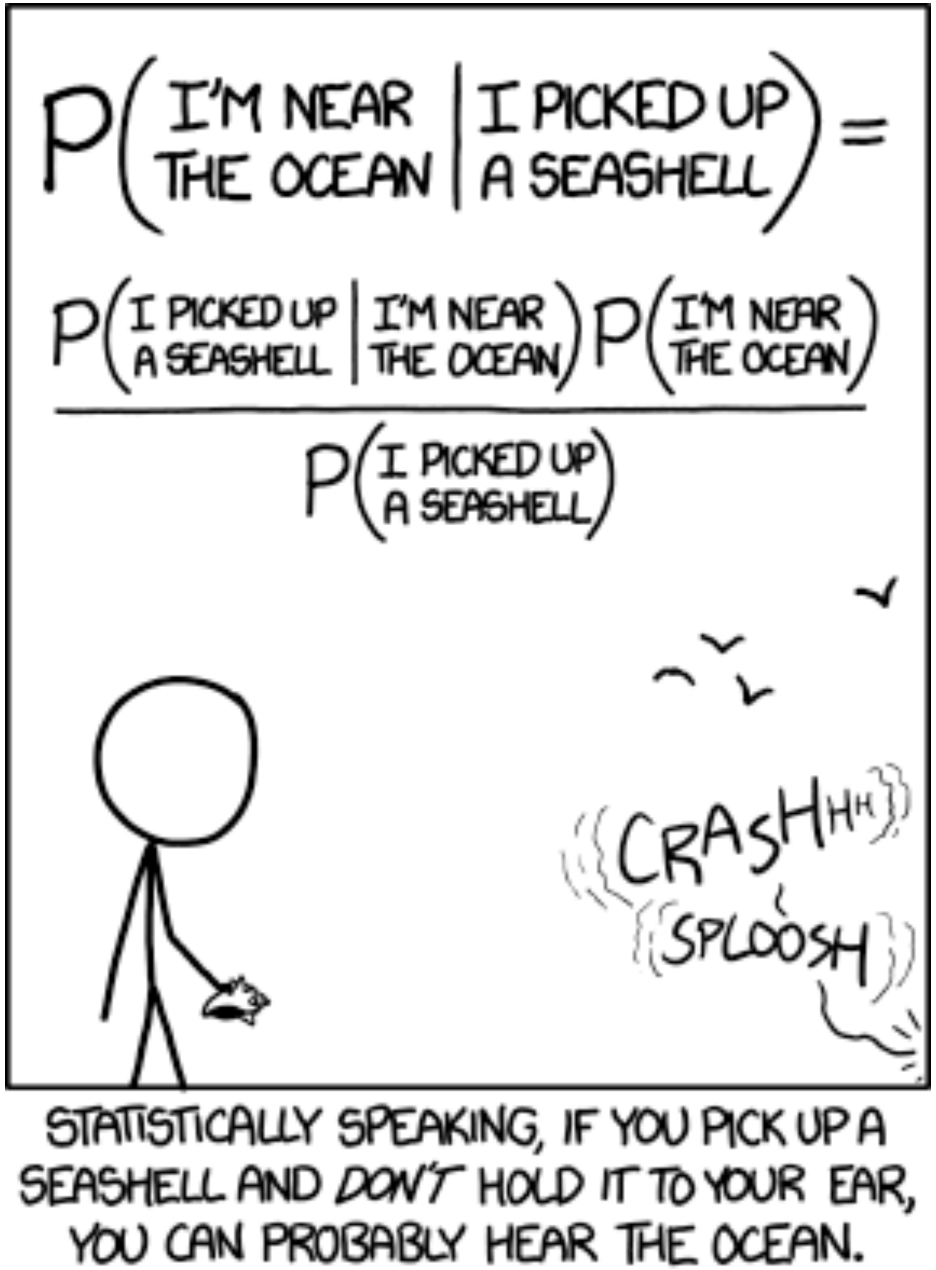
For e.g.,

disease | test result (we have estimates on test result | disease)

student knowledge | solved question (easier to estimate solve question | student knowledge)

spam | words (we'll see this ML example in detail in next week's section)

Remember, the evidence is almost always an estimate– we use Bayes' Theorem to **model** situations, which means we should always think about how good our model is.



https://xkcd.com/1236/

The Technical Stuff

Conditional Probability Defines a Probability Space

Probabilities conditioned on A follow the same properties as (unconditional) probability.

For example, $\mathbb{P}(B^c|A) = 1 - \mathbb{P}(B|A)$

Formally, $(\Omega, \mathbb{P})$ is a probability space and $\mathbb{P}(A) > 0$.

$\big(A, \mathbb{P}(.\,|A)\big)$ is a probability space.

Proof of Bayes' Rule

By definition of conditional probability,

$$\mathbb{P}(A \cap B) = \mathbb{P}(A|B)\mathbb{P}(B)$$
$$\mathbb{P}(B \cap A) = \mathbb{P}(B|A)\mathbb{P}(A)$$

Moreover, set intersection is commutative so $A \cap B = B \cap A$, and so, $\mathbb{P}(A \cap B) = \mathbb{P}(B \cap A)$.

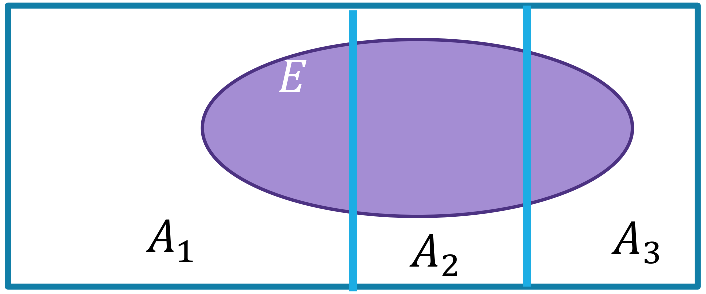
Then, we can see the following equation:

$$\mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A)$$

Dividing both sides by $\mathbb{P}(B)$, we get:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Formal proof of LTP



The Proof is actually pretty informative on what's going on.

$\sum_{\text{all } i} \mathbb{P}(E|A_i)\mathbb{P}(A_i)$

$= \sum_{\text{all } i} \frac{\mathbb{P}(E \cap A_i)}{\mathbb{P}(A_i)} \cdot \mathbb{P}(A_i)$ (definition of conditional probability)

$= \sum_{\text{all } i} \mathbb{P}(E \cap A_i)$

$= \mathbb{P}(E)$

The A_i partition Ω, so $E \cap A_i$ partition E. Then we just add up those probabilities.

Ability to add follows from the "countable additivity" axiom.

A Quick Technical Remark

I often see students write things like

$\mathbb{P}([A|B]|C)$

This is not a thing.

You probably want $\mathbb{P}(A|[B \cap C])$

$A|B$ isn't an event – it's describing an event **and** telling you to restrict the sample space. So, you can't ask for the probability of that conditioned on something else.