

Homework 1: Counting

For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will not receive full credit. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide.

In general, your goal in an explanation is to write enough that a student from class who has attended lecture, but not thought about the problem yet, could understand your approach, verify your reasoning, and believe your answer is correct. While we do not usually need to see arithmetic, you must include enough work that in principle one could rederive your answer with only a scientific calculator.

Unless a problem states otherwise, you should leave your answer in terms of factorials, combinations, etc., for instance 26^7 or $26!/7!$ or $26 \cdot \binom{26}{7}$ are all good forms for final answers.

Remember that you must “select pages” to identify which problem is on which page as part of your gradescope upload.

Due Date: This assignment is due at **11 PM Wednesday June 28**.

You will submit the written problems as a PDF to Gradescope. Please put each numbered problem on its own page of the PDF (this will make selecting pages easier when you submit), and ensure that your PDFs are oriented correctly (e.g. not upside-down or sideways). The coding problem will also be submitted to Gradescope.

Note that there will be **no leeway** with the submission time. Any submission to Gradescope after 11:00:00 PM on the deadline day will be considered late. We recommend submitting a few minutes before the deadline in order to avoid technical issues with submissions.

Collaboration: Please read the [full collaboration policy](#). If you work with others, you must still write up your solution independently and name all of your collaborators somewhere on your assignment.

Full submission instructions are in the course syllabus.

1. CSE Plays Soccer [15 points]

Twelve people (4 students and 8 faculty) on CSE’s soccer team show up for a game.

- (a) How many ways are there to choose which 4 players will play midfield positions (for this part, you should just account for who is a midfielder and who is not)? (5 points)
- (b) How many ways are there to assign the 4 midfield positions by selecting players from the 12 people who show up (for this part, account for both who plays and which of the 4 different positions they are playing)? (5 points)
- (c) How many ways are there to choose which 4 players will be midfielders if at least one of these players must be a student? (Go back to not deciding who plays which position for this part.) (5 points)

2. String cheese [24 points]

We want to count the number of strings of length 7 from the English alphabet $\{A, B, \dots, Z\}$ subject to a number of different constraints. Note that we consider the English alphabet here to consist of 6 vowels ($\{A, E, I, O, U, Y\}$) and 20 consonants. How many strings are there which...

- (a) (4 points) ...are only made of vowels?
- (b) (4 points) ...are only made of consonants?
- (c) (4 points) ...have *exactly* one vowel?

- (d) (4 points) ...have *exactly* two vowels?
- (e) (4 points) ...have at most two consonants, which may only appear in the sixth and seventh positions?
- (f) (4 points) ... have at least one consonant?

3. Red Riding Hood Visits Grandma [20 points]

- (a) How many paths are there from point (0,0) to (70,140) if every step increments one coordinate and leaves the other unchanged? (4 points)
- (b) How many paths are there from point (0,0) to (70,140) if every step increments one coordinate and leaves the other unchanged and you want the path to go through (20,50)? (4 points)
- (c) How many such paths are there from (0,0) to (70, 140) that go through (30,125), but do **not** go through (20,50)? (4 points)
- (d) How many paths are there from point (0,0) to (70,140) if every step increments one coordinate and leaves the other unchanged and the path **cannot** go through (20, 50) or (30,125)? (4 points)
- (e) How many paths are there from point (0,0,0) to (200,50,45) if every step increments one coordinate and leaves the other two unchanged? (4 points)

4. Three Peas Suit [10 points]

A new card game, Trio, is trending. Trio features a non-standard card deck of 42 cards. The deck has 3 suits, each of which has 14 ranks. How many 4-card hands are there, with at least one card from each suit?

5. A very useful engineering question [6 points]

How many different ways are there to arrange the letters in the word **engineering**?

6. Coding [15 points]

- (a) Read the [HW1 Python Tutorial and Coding Exercises](#) lesson on Edstem and follow the directions to complete 5 coding exercises. Then submit all required files to HW1 [Coding] on Gradescope. You may resubmit to gradescope as many times as you like. We do not have any hidden tests for this assignment; whatever score you see on gradescope on your last submission will be your final score. (10 points)
- (b) Read the [Edstem lesson](#) on Python's numpy library and write the reflection described below (be sure to finish part (a) before moving to this part). You do **not** need to complete the *coding* exercises for this part. The coding exercise that is there is optional, and intended only for practice if you want it. The goal of this part is to see some more advanced python features that will definitely be useful in the future, but you won't need to use frequently for this course.
After reading the lesson, write a reflection on what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort (since this response can't be "correct" or "incorrect"). We expect most responses will be 2-4 sentences. (5 points)