Lecture Topics: 1.3 No More Counting Please

[Tags: PSet1 Q7, Combinatorial Proofs]

1. **Give combinatorial proofs of the following identities:**
   a. Prove that \( \binom{n}{2} = \sum_{k=1}^{n-1} k \).
   b. Prove that \( 2^n - 1 = \sum_{i=0}^{n-1} 2^i \). (Hint: Imagine a tournament bracket)

[Tags: Complementary Counting, Inclusion-Exclusion, Multinomial Coefficients]

2. Find the number of ways to rearrange the word “INGREDIENT”, such that no two identical letters are adjacent to each other (there are two E’s, two N’s, and two I’s). For example, “INGREDINT” is invalid because the two E’s are adjacent. Repeat the question for the letters “AAAAAABBB”.

For more examples and solutions, see [https://courses.cs.washington.edu/courses/cse312/18sp/](https://courses.cs.washington.edu/courses/cse312/18sp/).