CSE 312: Foundations of Computing II
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Lecture Topics: 8.1/8.2 Confidence Intervals

[**Tags:** Confidence Intervals, Hypothesis Testing]

1. Suppose William claims that the true average height of students in the Allen School is indeed 72 inches (6 feet), but Cooper thinks it's anything but that! Cooper samples 145 students and finds the sample average height to be 63.41 inches and a sample variance of 6.32^2 sq. inches.
   a. First, compute a 98% confidence interval for the true average height μ, where our estimate was $\hat{\mu} = \bar{x} = 63.41$ inches.
   b. Conduct a hypothesis test following the procedure in 8.4 at the $\alpha = 0.02$ significance level. You can actually compute a confidence interval instead of computing a p-value when you have a two-sided alternative like we do here!

Solution:

a. Since $\hat{\mu} = \bar{x}$ is approximately normally distributed by the CLT, we can apply the formula:

$$[\hat{\theta} - \Delta, \hat{\theta} + \Delta]$$

where $\Delta = z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}$. We know that $100(1-\alpha) = 98 \to \alpha = 0.02$, so $z_{1-\alpha/2} = z_{0.99} = \Phi^{-1}(0.99) = 2.33$. We don't know σ but a good estimator for σ is $\hat{\sigma} = 6.32$ (square root of sample variance). Hence our

$$\Delta = 2.33 \cdot \frac{6.32}{\sqrt{145}} = 1.2229$$

$$[63.41 - 1.2229, 63.41 + 1.2229] = [62.1871, 64.6329]$$

b. We follow the procedure in the slides:

1. Make a claim.

The true average height of students in the Allen School is not 72 inches.

2. Set up the hypotheses.

$$H_0: \mu = 72 \qquad H_A: \mu \neq 72$$

3. Choose a significance level.

We are supposed to use $\alpha = 0.02$.

4. Collect data (already done).

5. Compute a p-value OR confidence interval.

Since the true assumed average height (under the null hypothesis) 72 inches is NOT in our 98% confidence interval $[62.1871, 64.6329]$, that means our p-value must be < 0.02 !

6. **State your conclusion.**

Since our 98% confidence interval does not contain the true hypothesized mean, we reject the null hypothesis. There is sufficient evidence to show that the true mean is different from 72 inches.

[**Tags:** Hypothesis Testing]

2. Luxi wants to perform a magic trick for her CSE 312 co-TA's Aleks and Shreya as an audience. She announces that she currently has a hat with 20 green marbles and 9 blue marbles. Aleks and Shreya do not trust her one bit, and think the number of blue marbles is actually an **underestimate** (but they believe that there are 20 green marbles).

   They reach in a grab 8 marbles randomly (without replacement), and 7 of them are blue! Conduct a hypothesis test following the procedure in 8.4 at the $\alpha = 0.05$ significance level to determine whether or not Luxi is lying about the number of blue marbles.

Solution:

We follow the procedure in the slides:

1. **Make a claim.**

There are more than 9 blue marbles in the bag.

2. **Set up the hypotheses.**

Let B denote the true number of blue marbles.

$$H_0: B = 9 \qquad\qquad H_A: B > 9$$

3. **Choose a significance level.**

We are supposed to use $\alpha = 0.05$.

4. **Collect data (already done).**

5. **Compute a p-value OR confidence interval.**

Our p-value is, the probability that when we draw 8 marbles from a bag, we get (at least) 7 blue marbles. The number of blue marbles we draw (under the null hypothesis) is $X \sim HypGeo(N = 29, K = 9, n = 8)$. Hence,

$$p\text{-value} = P(X \geq x) = P(X = 7) + P(X = 8) = \frac{\binom{9}{7}\binom{20}{1}}{\binom{29}{8}} + \frac{\binom{9}{8}\binom{20}{0}}{\binom{29}{8}} \approx 0.00017$$

6. State your conclusion.

Since our p-value $0.00017 < \alpha = 0.05$, we reject the null hypothesis. There is strong evidence to show that the true number of blue marbles is > 9.