CSE 312: Foundations of Computing II
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Lecture Topics: 7.3 Method of Moments Estimation, 7.4 Beta/Dirichlet Distributions

[**Tags:** Estimation]

1. Suppose $x = (x_1, \ldots, x_n)$ are iid samples from the following distributions. Estimate the parameter(s) using your favorite technique (MLE or MoM). **Hint**: Use MoM.
   a. The $Gamma(r, \lambda)$ distribution. Estimate both r and λ.
   b. The $Rayleigh(\sigma)$ distribution with density $f_X(x; \sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, x \geq 0$ with expectation $\sigma\sqrt{\frac{\pi}{2}}$.

[**Tags:** Beta/Dirichlet]

2. Suppose we roll a (possibly unfair) 4-sided die 29 times. Then, the number of times each digit appears is $\boldsymbol{X} = (X_1, X_2, X_3, X_4) \sim Mult_4(n = 29, \boldsymbol{p})$, where $\boldsymbol{p} = (p_1, p_2, p_3, p_4)$ is unknown. We happened to observe 5 ones, 7 twos, 6 threes, and 11 fours.
   a. A $Beta(\alpha_1, \beta_1)$ rv would be suitable to model our belief on p_1 (the probability of rolling a one) with what parameters α_1, β_1?
   b. A $Beta(\alpha_2, \beta_2)$ rv would be suitable to model our belief on p_2 (the probability of rolling a two) with what parameters α_2, β_2?
   c. Let's instead say we wanted to jointly model all the unknown parameters $\boldsymbol{p}$. A $Dirichlet(\boldsymbol{\gamma})$ would be suitable, more efficient than modelling all four separately, and also enforce that $\sum_{i=1}^{4} p_i = 1$. Which parameter vector $\gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ would best model our belief?