CSE 312: Foundations of Computing II
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Lecture Topics: 5.8 The Multinomial Distribution

[**Tags:** Multinomial, Multivariate Hypergeometric]

1. You work at a carnival and offer a simple game for children to play. The children get to choose a ticket from a bucket and based on the color of the ticket they will get different prizes. There are five colors of tickets: red, blue, green, and yellow which there are 24 of each, and gold which there are 4 of for a total of 100 tickets. Every day, 40 kids come to play the game. What is the probability that 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets are chosen on a given day if:
   a. Your boss is present, so you have to make the game fair and replace each ticket after it is chosen?
   b. Your boss isn't around, so you are lazy don't replace the tickets after each draw?

Solution:

a. This is a multinomial distribution since we have replacement so we have:

$$\boldsymbol{X} \sim Mult_5\left(n = 40, \boldsymbol{p} = \left(\frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{4}{100}\right)\right)$$

With X_1, X_2, X_3 and X_4 describing the number of red, blue, green, and yellow tickets pulled respectively and X_5 describing the number of gold tickets pulled.
We have $\boldsymbol{p} = \left(\frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{4}{100}\right)$ so p_1, p_2, p_3 and p_4 describe the probability of red, blue, green, and yellow tickets being pulled and p_5 describes the probability of gold tickets pulled.

Now, we want to solve for the probability of 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets being pulled and we have:

$$p_{X_1,X_2,X_3,X_4,X_5}(9,9,9,9,4) = \binom{40}{9,9,9,9,4}\left(\frac{24}{100}\right)^{9+9+9+9}\left(\frac{4}{100}\right)^4 = 0.000244$$

b. Now, since we are without replacement, we have a multivariate hypergeometric and have:

$$\boldsymbol{X} \sim MVHG_5(N = 100, \boldsymbol{K} = (24,24,24,24,4), n = 40)$$

With X_1, X_2, X_3 and X_4 describing the number of red, blue, green, and yellow tickets pulled respectively and X_5 describing the number of gold tickets pulled.
We have $\boldsymbol{K} = (24,24,24,24,4)$ so k_1, k_2, k_3 and k_4 describe the initial number of red, blue, green, and yellow tickets and k_5 describes the initial number of gold tickets.
Now, we want to solve for the probability of 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets being pulled and we have:

$$p_{X_1,X_2,X_3,X_4,X_5}(9,9,9,9,4) = \frac{\binom{24}{9}\binom{24}{9}\binom{24}{9}\binom{24}{9}\binom{4}{4}}{\binom{100}{40}} = 0.000213$$