Lecture Topics: 5.4 Covariance and Correlation

[Tags: Covariance]

1. The covariance matrix of a random vector $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_n)$ is defined to be the $n \times n$ matrix $\Sigma$ such that $\Sigma_{ij} = Cov(X_i, X_j)$. Don’t be too intimidated - it’s just a way to store all the information we need – we won’t be doing any linear algebra with it! The examples will help ☺.

$$
\Sigma = 
\begin{bmatrix}
Cov(Z_1, Z_1) & Cov(Z_1, Z_2) & \cdots & Cov(Z_1, Z_n) \\
Cov(Z_2, Z_1) & Cov(Z_2, Z_2) & \cdots & Cov(Z_2, Z_n) \\
\vdots & \vdots & \ddots & \vdots \\
Cov(Z_n, Z_1) & \cdots & \cdots & Cov(Z_n, Z_n)
\end{bmatrix}
$$

a. Let $X_1, X_2, X_3, X_4$ be iid (independent and identically distributed) random variables with mean $\mu$ and variance $\sigma^2$. Let $\mathbf{X} = (X_1, X_2, X_3, X_4)$ be a random vector with the rvs $X_i$ as its components. What is the $(4 \times 4)$ covariance matrix of $\mathbf{X}$?

b. Define $\mathbf{Y} = (Y_1, Y_2, Y_3)$ as follows.

- $Y_1 = X_1 + X_2$
- $Y_2 = X_2 + X_3$
- $Y_3 = X_3 + X_4$.

What is the $(3 \times 3)$ covariance matrix of $\mathbf{Y}$?

[Tags: Similar to PSet4 Q3, Covariance]

2. Suppose we throw 12 balls independently and uniformly into 7 bins. For $i = 1, \ldots, 7$, let $X_i$ be the indicator/Bernoulli rv of whether bin $i$ is empty. Let $\mathbf{X} = (X_1, \ldots, X_7)$ be the random vector of indicators.

a. What is the covariance matrix of $\mathbf{X}$?

b. Let $Y = \sum_{i=1}^7 X_i$ be the number of empty bins. What is $Var(Y)$?