

CSE 312: Foundations of Computing II

Instructor: Alex Tsun

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Lecture Topics: 4.2 Zoo of Continuous Random Variables

[Tags: PDFs, CDFs, Exponential, Uniform]

1. You are waiting for a bus to take you home from CSE. You can either take the E-line, U-line, or C-line. The distribution of the waiting time in minutes for each is the following:
 - E-Line: $E \sim \text{Exp}(\lambda = 0.1)$
 - U-Line: $U \sim \text{Unif}(0, 20)$ (continuous)
 - C-line: Has range $(1, \infty)$ and density function $f_C(x) = 1/x^2$.

Assume the three bus arrival times are independent. You take the first bus that arrives.

- a. Find the CDF's of E , U , and C , $F_E(t)$, $F_U(t)$, and $F_C(t)$. **Hint:** The first two can be looked up in a table.
- b. What is the probability you wait more than 5 minutes for a bus?
- c. What is the probability you wait more than 30 minutes for a bus?
- d. (Challenge) What is the expected amount of time you will wait for a bus? **Hint:** Compute the CDF first which has four parts: $(-\infty, 0]$, $(0, 1]$, $(1, 20]$, and $(20, \infty)$.

[Tags: PSet3 Q5, Exponential, Memorylessness, Gamma]

2. You have n batteries, each with a lifetime which is (independently) distributed as $\text{Exp}(\lambda)$. You have a choice of a weak flashlight, which requires one battery to operate, and a strong flashlight, which requires two batteries to operate. Assume that when a battery dies, you are lightning-quick and replace it with a new battery instantly.
 - a. If you choose to use the weak flashlight, what is the expected amount of time you can operate it for? (Hint: Cite the appropriate distribution, and your solution will be one-line.)
 - b. Recall the memoryless property in lecture 4.2. Suppose $W \sim \text{Exp}(\beta)$. Show that you understand what it means by computing $P(W > 17 | W > 10)$ explicitly using this property (do NOT reprove memorylessness).
 - c. For the strong flashlight, we need to compute the distribution of time that until the first of the two batteries dies. If $X, Y \sim \text{Exp}(\lambda)$, show that the distribution of $Z = \min\{X, Y\}$ is $\text{Exp}(2\lambda)$. (Hint: Start by computing $P(Z > z)$, then use this to compute either the CDF or PDF).
 - d. Left for you!