CSE 312: Foundations of Computing II

Section 6: Joint Distributions

1. Random Stick

You hold a stick of unit length (1). Someone comes along and breaks off a random piece at some point $Y \sim \text{Unif}(0, 1)$. Now you hold a stick of length Y. Another person comes along and breaks off another piece from the remaining part of the stick that you hold at point $X \sim \text{Unif}(0, Y)$. You are left with a stick of length X. Find the PDF $f_X(x)$, mean $\mathbb{E}[X]$ **using LTE** and variance $\text{Var}(X)$ **using LTE as well**.

2. Another Urn Question

An urn has 12 balls, 5 red ones and 7 green ones. Draw 3 balls. Let X denote the number of red balls in the sample. Compute $Var(X)$ when sampling is done:

(a) With replacement

(b) Without replacement

3. Continuous Joint Density

The joint probability density function of X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & 0 < x < 1,\ 0 < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Verify that this is indeed a joint density function.

(b) Compute the marginal density function of X.

(c) Find $P(Y > \frac{1}{2} | X < \frac{1}{2})$.

(d) Find $E(X)$.

(e) Find $E(Y)$