

PSet #1

With problems from several past UW CSE 312 instructors (Martin Tompa, Anna Karlin, Larry Ruzzo) and Stanford CS 109 instructors (Chris Piech, David Varodayan, Lisa Yan, Mehran Sahami)

Groups: The written part of this PSet may be done in groups of **up to 2 people**. This means that only one person will submit on Gradescope to “PSet 1 [Written]” and add their partner as a collaborator. The coding part must be done individually, so each student will submit their own coding assignment (and associated written response) to “PSet 1 [Coding]” and “PSet 1 [Q13 Written]” respectively. Individuals and groups are encouraged to discuss problem-solving strategies with other classmates as well as the course staff, but student’s submissions must be their own work.

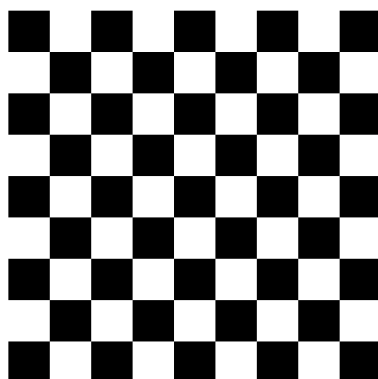
Instructions: For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. It is fine for your answers to include summations, products, factorials, exponentials, or combinations; you don’t need to calculate those all out to get a single numeric answer.

Submission: You must upload your written compiled LaTeX PDF to Gradescope under “PSet 1 [Written]” (with your partner if applicable), your two code files [pingpong.py](#) and [pokemon.py](#) to “PSet1 [Coding]”, and your solution to question 13’s written component to “PSet 1 [Q13 Written]”. Extra credit can be submitted by groups or individuals in their “PSet 1 [Written]” and any code for it should be included in the submitted PDF. You must tag your written problems on Gradescope, or you will receive **no credit** as mentioned in the syllabus. Please cite any collaboration at the top of your submission (beyond your group members, which should already be listed).

1. How many ways can 18 people be seated in a row if ...
 - (a) ...there are no restrictions on the seating arrangement?
 - (b) ...two of the people, persons A and B, cannot sit (immediately) next to each other?
 - (c) ...there are 9 adults and 9 children, and no two adults nor two children can sit next to each other?
 - (d) ...there are 9 married couples and each couple must sit together?

2. At the local zoo, a new exhibit consisting of 5 different species of fish and 5 different species of primates is to be formed from a pool of 10 fish species and 9 primate species. How many exhibits are possible if ...
 - (a) ... there are no additional restrictions on which species can be selected?
 - (b) ... 2 particular fish species cannot both be in the exhibit (e.g., they have a predator-prey relationship)?

- (c) ...any fish species can be in the exhibit, but 1 particular fish species cannot be placed with 1 particular primate species?
3. A piano octave consists of 12 notes in ascending order. Five of them are black key notes and seven are white key notes. (See <http://www.smackmypitchup.com/smpu/content/img/MT/mtp01.gif> for a picture; the notes are ascending from left to right.) A composer is experimenting with the idea that a melody must be a sequence of 6 notes from this single octave, 2 of them black and 4 of them white. The notes of a melody need not be distinct: you can use the same note 2 or more times. How many possible melodies are there if . . .
- (a) . . . there are no further restrictions?
- (b) . . . the 4 white notes cannot *all* be adjacent in the melody? (“Adjacent” here does not mean adjacent on the keyboard, but rather occurring without any intervening black notes in the melody). For example, the pattern [WWWWBB] is *not* allowed but [WWWBWB] *is* allowed.
- (c) . . . no note is allowed to be repeated?
- (d) . . . no note is allowed to be repeated and the white notes must be ascending in the melody?
4. Say a hacker has a list of n distinct password candidates, only one of which will successfully log her into a secure system.
- (a) If she tries passwords from the list at random, deleting those passwords that do not work, what is the probability that her first successful login will be (exactly) on her k -th try? Assume $k \leq n$.
- (b) Now say the hacker tries passwords from the list at random, but does not delete previously tried passwords from the list (she may try the same password multiple times). She stops after her first successful login attempt. What is the probability that her first successful login will be (exactly) on her k -th try?
5. Each square of a 9x9 checkerboard (see below) is initially slept on by one of 81 students. At noon, each student will wake up and randomly sleepwalk to a valid adjacent square horizontally or vertically (but not diagonally). Argue that the probability that two or more students end up on the same square is 1.



6. For each of the following scenarios, a proposed answer is given. If the answer is correct, say so. If we undercounted, describe what we undercounted and how to fix it. If we overcounted, describe what we overcounted and how to fix it.

- (a) The number of ways to arrange 7 people evenly spaced around a circular table is: $7!$. (We do not count equivalent rotations as different arrangements.)
- (b) Suppose there are 8 different toppings I can choose for my pizza. Each topping is either on my pizza or not. Then, the number of ways I can choose 3 **different** pizzas, one for each of my three children is: $(2^8)(2^8 - 1)(2^8 - 2)/3!$.
- (c) The number of ways to distribute eleven indistinguishable pizzas to my 3 siblings and six indistinguishable burgers to my 4 grandparents is: $\binom{14}{3} \cdot \binom{10}{4}$.
- (d) There are 3 email servers which are all initially empty. 312 unique emails arrive, and each email is randomly routed to one of the 3 servers, independently. The number of ways can the emails be distributed so that none of the servers are empty is: $3^{312} - \binom{3}{1}2^{312}$.

7. Give combinatorial proofs of the following identities:

- (a) $\binom{n}{2} = \sum_{k=1}^{n-1} k$.
- (b) $2^n - 1 = \sum_{i=0}^{n-1} 2^i$. (Hint: Imagine a tournament bracket.)

8. Suppose you went trick-or-treating (as an adult) and were able to nab N total candies, K of which are kit-kats. The following two parts should be treated **separately** - only the information above is common to both.

- (a) Your responsible parent says you can only eat n of them tonight. You reach in and randomly grab n of them. Let X be the number of kit-kats you grabbed. What is $P(X = k)$ for valid values of k ?
- (b) You eat candies one at a time, randomly without looking into the bag, and decide to stop when you've eaten k kit-kats. Let Y be the total number of candies you ate, up to and including the time you ate your k -th kit-kat. What is $P(Y = n)$ for valid values of n ?

9. Each of 100 students in the class can only take 1 computer science class each, between the four classes `discrete-math`, `intro-programming`, `data-structures`, and `computer-architecture`. Each student (independently of others) takes `discrete-math` with probability 0.3, `intro-programming` with probability 0.4, `data-structures` with probability 0.1, and `computer-architecture` with probability 0.2. What is the probability that *all* of the following conditions are satisfied simultaneously?

- 29 sign up for `discrete-math`
- 41 sign up for `intro-programming`
- 6 sign up for `data-structures`
- and 24 sign up for `computer-architecture`?

10. A website wants to detect if a visitor is a robot or a human. They give the visitor seven CAPTCHA tests that are hard for robots but easy for humans. If the visitor fails any of the tests, they are flagged as a robot. The probability that a human succeeds at a single test is 0.96, while a robot only succeeds with probability 0.2. Assume all tests are independent. The percentage of visitors on this website that are robots is 10%; all other visitors are human.
- (a) If a visitor is actually a robot, what is the probability they get flagged (the probability they fail at least one test)?
 - (b) If a visitor is actually a human, what is the probability they get flagged (the probability they fail at least one test)?
 - (c) Suppose a visitor gets flagged. What is the probability that the visitor is a robot? **Give your answer to 4 decimal places.**
11. There are 4 decks of cards: a red deck (52 cards), a green deck (104 cards), a blue deck (156 cards), and a yellow deck (208 cards). A standard 52-card deck consists of one card of each suit-rank combination (there are 4 suits and 13 ranks). The red deck is a standard 52-card deck, the green deck consists of 2 standard 52-card decks, the blue deck consists of 3 standard 52-card decks, and the yellow deck consists of 4 standard 52-card decks.
- (a) We draw from a deck with probability proportional to the number of cards in that deck (e.g., we are three times as likely to choose from the blue deck than the red deck). Give the probabilities of drawing from each deck. Your answer should be 4 probabilities that sum to 1.
 - (b) If we draw three cards from the blue deck **without** replacement, what is the probability of observing the sequence (King of Hearts, Ace of Spades, King of Hearts) in this order?
 - (c) Given we observed the sequence (King of Hearts, Ace of Spades, King of Hearts) in this order while drawing from a random deck **without** replacement, what is the probability we drew from the blue deck? **Give your answer to 4 decimal places.**
12. A home security system may detect movement using its two different sensors. If motion is detected by any of the sensors, the system will alert the police. If there is movement outside, sensor V (video camera) will detect it with probability 0.95, and sensor L (laser) will detect it with probability 0.8. If there is no movement outside, sensor L will detect motion anyway with probability 0.05, and sensor V will detect motion anyway with probability 0.1. Based on past history, the probability that there is movement at a given time is 0.7. Assume these sensors have proprietary algorithms, so that **conditioned** on there being movement (or not), the events of detecting motion (or not) for each sensor is **independent**.
- (a) Given that there is movement outside **and** that sensor V does not detect motion, what is the probability that sensor L detects motion?
 - (b) Given that there is a moving object, what is the probability that the home security system alerts the police?
 - (c) What is the probability of a false alarm? That is, that there is no movement but the police are alerted anyway?

(d) What is the probability that there is a moving object given that both sensors detect motion?

13. **[Coding+Written]** We'll finally answer the long-awaited question: what's the probability you win a ping pong game up to n points, when your probability of winning each point is p (and your friend wins the point with probability $1 - p$)? Assume you have to win by (at least) 2; for example, if $n = 21$ and the score is 21 : 20, the game isn't over yet. (e.g., a score could go to 26 : 24 if necessary, but it could also end 21 : 13).

Write your code for the following parts in the provided file: [pingpong.py](#).

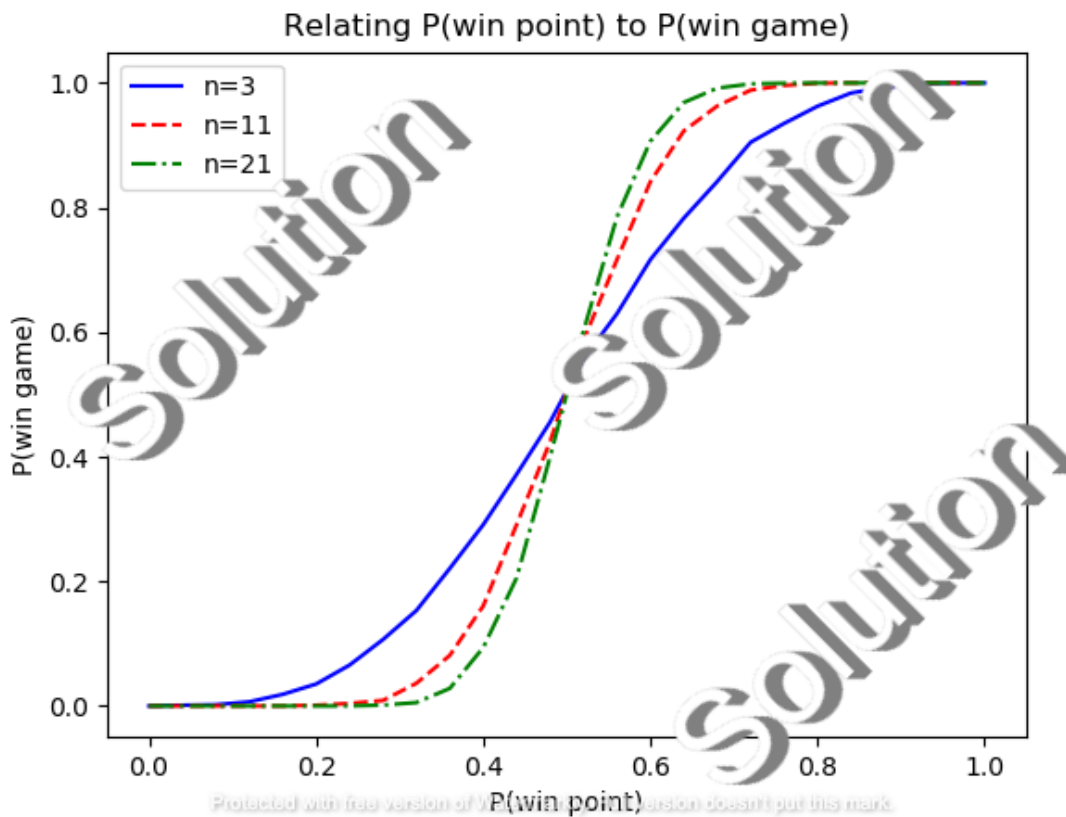
- (a) Implement the function [part_a](#).
- (b) Implement the function [part_b](#). This function will NOT be autograded but you will still submit it; you should use the space here to generate the plot asked of you below.
 - i. Generate the plot below in Python (without the watermarks). Details on how to construct it are in the starter code. Attach your plot in your written submission for this part.
 - ii. Write AT MOST 2-3 sentences identifying the interesting pattern you notice when n gets larger (regarding the steepness of the curve), and explain why it makes sense.
 - iii. Each curve you make for different values of n always (approximately) passes through 3 points. Give the three points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and explain why mathematically this happens in AT MOST 2-3 sentences.

14. **[Coding]** Let's learn how to use Python and data to do approximate quantities that are hard to compute exactly! By the end of this, we'll see how long it actually takes to "catch'em all"! You are given a file [pokemon.txt](#) which contains information about several (fictional) Pokemon, such as their encounter rate and catch rate.

Write your code for the following parts in the provided file: [pokemon.py](#).

- (a) Implement the function [part_a](#).
 - (b) Implement the function [part_b](#).
 - (c) Implement the function [part_c](#).
 - (d) Implement the function [part_d](#).
15. **(Extra Credit)**: If you worked with a partner that you were randomly paired with during a social event or through the partner survey, attach a screenshot here to get extra credit! If it was a social event zoom call, your screenshot must include the zoom meeting information to prove it was one of our social zoom meetings.
16. **(Extra Credit) [Coding+Written]**: Calculate, by Python simulation, a probability or average that you're interested in, but never knew how to compute! Describe the problem concisely in words, and provide enough background information if necessary so that someone not familiar with it can understand. Then, write Python code to simulate it! Does the answer surprise you? Include some kind of result here; whether it be numerical or a plot, as well as your code (you don't have to submit .py files but we should be able to use your code submitted in this pdf to recreate your results).

Figure 1: Your plot should look something like this.



17. **(Extra Credit) [Written]**: Consider the ping pong scenario in problem 13. For $n = 21$, compute the **exact** probability of winning a game, as a function of the probability of winning a single point p . Your answer may include summations and binomial coefficients. Then, evaluate your answer when $p = 0.3$ and give your answer to 6 decimal places. (Hint: Consider two cases; one where your final score was 21, and one where you had to play until you won by 2.)