CSE 312: Foundations of Computing II

Section 1: Counting I

1. Summary of Main Concepts (Counting I)

(a) **Sum Rule:** If an experiment can either end up having one of N outcomes, or one of M outcomes (where there is no overlap), then the total number of possible outcomes is $N + M$.

(b) **Product Rule:** If an object or an outcome can be selected by a sequence of choices, where there are m_1 possibilities for the first choice, m_2 possibilities for the second choice (given the first), m_3 possibilities for the third choice (given the first two) and so on up to the k-th choice, then there are $m_1 \cdot m_2 \cdot m_3 \cdots m_k = \prod_{i=1}^{k} m_i$ possible outcomes overall.

(c) **Number of ways to order n distinct objects:** $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$

(d) **Complementary Counting (Complementing):** If asked to find the number of ways to do X, you can: find the total number of ways and then subtract the number of ways to not do X.

2. Sets

a) For each one of the following sets, give its cardinality, i.e., indicate how many elements it contains:

- $A = \emptyset$
- $B = \{\emptyset\}$
- $C = \{\{\emptyset\}\}$
- $D = \{\emptyset, \{\emptyset\}\}$

b) Let $S = \{a, b, c\}$ and $T = \{c, d\}$. Compute:

- $S \cup T$
- $S \cap T$
- $S \setminus T$
- $2^{S \setminus T}$
- $S \times T$

3. Basic Counting

a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is *uniquely determined* by the first 15 digits). How many credit-card numbers are there?

b) How many positive divisors does $1440 = 2^5 3^2 5$ have?

c) How many ways are there to arrange the CSE 312 staff on a line (Aleks and 6 TAs) for a group picture?

d) There are 50 seats and 50 people (44 students, 6 TAs) in the classroom. Suppose that the back row contains exactly 6 seats and the 6 TAs must sit in the back row. How many seating arrangements are possible with this restriction?

4. Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

(a) . . . all couples are to get adjacent seats?

(b) . . . anyone can sit anywhere, except that one couple insists on *not* sitting in adjacent seats?

5. Weird Card Game

In how many ways can a pack of fifty-two cards be dealt to thirteen players, four to each, so that every player has one card of each suit?

6. Birthday Cake

A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

7. Photographs

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

8. Rabbits!

Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

9. Chickens and More Rabbits!

Our class has three pet rabbits, Flopsy, Mopsy, and Cottontail, as well as two pet chickens, Ginger and Rocky, but only 4 shelters to put them in. In how many different ways can we put these 5 pets in these 4 shelters so that no shelter has both a chicken and a rabbit?

10. Extended Family Portrait

A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?