CSE 312: Foundations of Computing II

Section 1: Counting I

1. Summary of Main Concepts (Counting I)
   (a) **Sum Rule:** If an experiment can either end up having one of \( N \) outcomes, or one of \( M \) outcomes (where there is no overlap), then the total number of possible outcomes is \( N + M \).

   (b) **Product Rule:** If an object or an outcome can be selected by a sequence of choices, where there are \( m_1 \) possibilities for the first choice, \( m_2 \) possibilities for the second choice (given the first), \( m_3 \) possibilities for the third choice (given the first two) and so on up to the \( k \)-th choice, then there are \( m_1 \cdot m_2 \cdot m_3 \cdots m_k = \prod_{i=1}^{k} m_i \) possible outcomes overall.

   (c) **Number of ways to order \( n \) distinct objects:** \( n! = n \cdot (n - 1) \cdots 3 \cdot 2 \cdot 1 \)

   (d) **Complementary Counting (Complementing):** If asked to find the number of ways to do \( X \), you can: find the total number of ways and then subtract the number of ways to not do \( X \).

2. Sets
   a) For each one of the following sets, give its cardinality, i.e., indicate how many elements it contains:
      - \( A = \emptyset \)
      - \( B = \{\emptyset\} \)
      - \( C = \{\{\emptyset\}\} \)
      - \( D = \{\emptyset, \{\emptyset\}\} \)

   b) Let \( S = \{a, b, c\} \) and \( T = \{c, d\} \). Compute:
      - \( S \cup T \)
      - \( S \cap T \)
      - \( S \setminus T \)
      - \( 2^{S \setminus T} \)
      - \( S \times T \)

3. Basic Counting
   a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is uniquely determined by the first 15 digits). How many credit-card numbers are there?

   b) How many positive divisors does \( 1440 = 2^5 \cdot 3^2 \cdot 5 \) have?

   c) How many ways are there to arrange the CSE 312 staff on a line (Aleks and 6 TAs) for a group picture?

   d) There are 50 seats and 50 people (44 students, 6 TAs) in the classroom. Suppose that the back row contains exactly 6 seats and the 6 TAs must sit in the back row. How many seating arrangements are possible with this restriction?

4. Seating
   How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .
   (a) . . . all couples are to get adjacent seats?

   (b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?
5. **Weird Card Game**
In how many ways can a pack of fifty-two cards be dealt to thirteen players, four to each, so that every player has one card of each suit?

6. **Birthday Cake**
A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

7. **Photographs**
Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

8. **Rabbits!**
Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

9. **Chickens and More Rabbits!**
Our class has three pet rabbits, Flopsy, Mopsy, and Cottontail, as well as two pet chickens, Ginger and Rocky, but only 4 shelters to put them in. In how many different ways can we put these 5 pets in these 4 shelters so that no shelter has both a chicken and a rabbit?

10. **Extended Family Portrait**
A group of \(n\) families, each with \(m\) members, are to be lined up for a photograph. In how many ways can the \(nm\) people be arranged if members of a family must stay together?