Problem Set 1 (due Friday, July 1 at 11:59pm)

Directions: For each problem, explain/justify how you obtained your answer, as correct answers without an explanation may receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. Unless you are asked to, you should leave your answer in terms of factorials, combinations, etc., for instance 26^7 or 26!/7! or $26 \cdot \binom{26}{7}$.

Submission: You must upload a **pdf** of your solutions to Gradescope under "Pset 1 [Written]". The use of LaTex is highly recommended, and we have provided a template. (Note that if you want to hand-write your solutions, you'll need to scan them. If we cannot make out your writing, your work may be ungradable, so make sure it is legible.) Your code will be submitted as a .py file under "Pset 1 [Coding]".

Instructions as to how to upload your solutions to Gradescope are on the course web page.

Remember that you must tag your written problems on Gradescope, or you will potentially receive **no credit** as mentioned in the syllabus. Please put each numbered problem on its own page in the pdf (this will make selecting pages easier when you submit), and ensure that your pdfs are oriented correctly (e.g. not upside-down or sideways). As stated above, the coding problem will also be submitted to Gradescope.

Collaboration: This pset must be submitted **individually**. You are welcome and encouraged to discuss approaches with your fellow students, but everyone **must write up their own solutions**. Failure to do so is an instance of academic dishonesty.

Individuals and pairs are still encouraged to discuss problem-solving strategies with other classmates as well as the course staff, but each pair must write up their own solutions. There will be a partner-finding form that you can **optionally** use if you'd like help finding a partner. You may also work individually if you'd like.

1. Settling Down (10 points)

Let us define the universe U to be all natural numbers, $S = \{4,7,10\}$, $A = \{1,2,5\}$, and $B = \{2,5\}$. For each of the following questions, determine if the statement is true or false. **Explain** your reasoning in one or two brief sentences.

- (a) [1 Point] $\{1\} \in A$
- (b) [1 Point] $B \subseteq A$
- (c) [1 Point] $\emptyset \in S$
- (d) [1 Point] A = B
- (e) [1 Point] $A = (A \cup B)$
- (f) [1 Point] $\emptyset = S \cap A$
- (g) [1 Point] $1 = A \setminus B$
- (h) [1 Point] $A^C \neq B^C$
- (i) [1 Point] $\emptyset \in U$
- (j) [1 Point] $\{2,3,5\} = (A \cap B) \cup \{3\}$

2. Summing It Up (10 points)

For each of the following problems assume that you have 3 skittles (1 red, 1 orange, 1 green), 4 m&ms (1 red, 1 green, 1 blue, 1 brown), and 2 starbursts (1 yellow, 1 pink).

- (a) [3 Points] How many ways are there to eat exactly 1 skittle, 1 m&m, and 1 starburst?
- (b) [4 Points] How many ways are there to eat 1 starburst, and exactly one other candy that must be either a skittle or an m&m?
- (c) [4 Points] How many ways are there to eat exactly 1 skittle and 1 m&m such that they are NOT the same color?

3. Softball (10 points)

For each of the following problems, assume that you are forming a softball team out of ten candidates (7 children and 3 adults).

- (a) [3 Points] How many ways are there to choose 4 players to be part of the team?
- (b) [3 Points] How many ways are there to assign 4 players to the positions of catcher, pitcher, 1st baseman and shortstop by selecting players from the 10 people who show up?
- (c) [4 Points] How many ways are there to choose 4 players to be part of the team if at least one of these players must be an adult?

4. Sitting around (15 points)

Archer (A), Bilbo (B), Cersei (C), Dante (D), Eowyn (E), Frodo (F), and Gollem (G) are sitting in a row of nine seats (Note: there are only seven people). Archer and Bilbo are exes, so they cannot sit next to each other. Cersei and Dante are dating, so they must sit next to each other. Eowyn, Frodo, and Gollem are best friends, so they also must sit next to each other, but Frodo must be in the middle of Eowyn and Gollem (with no spaces between the three). Our goal is to figure out how many ways they can sit in a row. Build up to the answer by answering the following questions.

Hint: We will start by grouping Eowyn, Frodo, and Gollem, as well as Cersei and Dante. Then we will work on placing Archer and Bilbo. Also, you will find the problem easier if you just call them, A,B,C,D, E, F, and G, as we do henceforth.

- (a) [3 Points] How many ways are there to place the 7 people into the 9 chairs if EFG must sit together in that order and CD must sit together in that order?
 - Hint: This is not unlike the rearrangements of DOGGY that we discussed/will discuss in lecture (where the empty seats are like the two Gs.)
- (b) [4 Points] How many ways are there to place the 7 people into the 9 chairs if EFG must sit together but E and G can swap positions and CD must sit together in either order?
- (c) [4 Points] How many ways are there to place the 7 people into the 9 chairs if EFG must sit together (but E and G can swap positions), CD must sit together in either order and AB must sit together in either order?

(d) [4 Points] How many ways are there to place the 7 people into the 9 chairs if EFG must sit together (but E and G can swap positions), CD must sit together in either order and A and B must **not** sit next to each other?

5. Coding+Reflection (15 points)

- (a) [10 Points] Read the Pset1 Coding lesson on Edstem and follow the directions to complete 5 coding exercises. Then submit all required files to PSet1 [Coding] on Gradescope. The score that appears on Gradescope for this part is final.
- (b) [5 Points] Read the Edstem lesson on Python's numpy library, after completing the previous part. You do **not** need to complete any coding exercises or submit anything to Gradescope for this part. The exercise that is there is entirely OPTIONAL, and intended only for practice if you need it. Afterwards, write down what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort rather than correctness, and it's recommended that your answer be no longer than 5 sentences.