Instructors

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A Team of fantastic TAs

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Chloe Winston
Claris Winston
Ben Zhang
Lectures and Sections

• **Lectures MWF**
  – 9:30-10:20am (Stefano) or 1:30-2:20pm (Paul)
  – Panopto recording and video released after class
  – Annotated slides also uploaded.

• **Poll Everywhere**
  – Classes will be in person
  – We will sometimes use Poll Everywhere during class
  – As of this quarter it requires that you sign up directly

• **Sections Thu (starts this week)**
  – Not recorded
Questions and Discussions

• **Office hours throughout the week (starting this Friday)**

• **Ed Discussion**
  
  You should have received an invitation (synchronized with the class roaster)
  
  – Material (resources tab)
  – Announcements (discussion tab)
  – Discussion (discussion tab)

  Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructors for personal issues.
Engagement

• “Concept checks” after each lecture 5-8 %
  – Must be done (on Gradescope) before the next lecture by 9:00am
  – Simple questions to reinforce concepts taught in each class
  – Keep you engaged throughout the week, so that homework becomes less of a hurdle

• 8 Problem Sets (Gradescope) 45-50 %
  – Solved individually. Discussion with others allowed but solution must be produced completely separately.
  – Generally due Wednesdays starting next week
  – First probm set posted this week before section

• Midterm 15-20 %
  – In class on Wednesday, May 4

• Final Exam 30-35 %
  – Monday, June 6 either 2:30-4:20 or 4:30-6:20 location TBA
  – For the A section this is a different time from the one on the UW final exam timetable

Check out the syllabus for policies on late submission for checkpoints and HW
Sections

• Sections will start this Thursday
• Will serve as additional preparation for problem set
• Attend them!
More details see

Course Webpage  https://cs.washington.edu/312
Foundations of Computing II

= Introduction to Counting, Probability & Statistics

for computer scientists

What is probability??
Why probability?!
Probability is our tool for understanding random processes

• Randomness in nature and the sciences/social sciences
  – At the quantum level, everything is random
  – Best way to understand and simulate behavior of complex systems
  – A way to design and understand experiments, observations
    • In the lab, the field, medical trials, surveys

• In Computer Science, randomness has these kinds of roles but also important new ones...
Probability and randomness in Computer Science

• Understanding/modelling the inputs to and behavior of our algorithms
  – In ML, program testing/fuzzing, algorithm analysis, ...

• Experiments to validate our designs
  – In user studies, HCI, CS applications in other fields, ...

• A tool for hiding information, protecting against adversaries/failures
  – Cryptography, privacy, fault tolerance, computer security, ...

• A tool for simpler and more efficient design
  – Data structures, algorithms, ML, ...

• ...

Content

• Counting (basis of discrete probability)
  – Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle

• What is probability
  – Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance

• Properties of probability
  – Various inequalities, Zoo of discrete random variables, Concentration, Tail bounds

• Continuous Probability
  – Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation

• Applications
  – A sample of randomized algorithms, differential privacy, learning ...
Today: Counting
We are interested in counting the number of objects with a certain given property.

“How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?”

“How many integer solutions \((x, y, z) \in \mathbb{Z}^3\) does the equation \(x^3 + y^3 = z^3\) have?”

Generally: Question boils down to computing cardinality \(|S|\) of some given set \(S\).
(Discrete) Probability and Counting are Twin Brothers

“What is the probability that a random student from CSE312 has black hair?”

\[
\frac{\# \text{ students with black hair}}{\# \text{students}}
\]
Sum Rule

If you can choose from

- EITHER one of $n$ options,
- OR one of $m$ options with NO overlap with the previous $n$

then the number of possible outcomes of the experiment is

$$n + m$$
Counting “lunches”

If a lunch order consists of either one of 6 soups or one of 9 salads, how many different lunch orders are possible?
**Product Rule:** In a sequential process, there are
- $n_1$ choices for the first step,
- $n_2$ choices for the second step (given the first choice), ..., and
- $n_m$ choices for the $m$th step (given the previous choices),
then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$

**Example:** “How many subways?”

$n_1 = 2$ cheeses

$\times$

$n_2 = 3$ meats

$\times$

$n_3 = 3$ veggies

$\times$
Product rule example – Strings

How many string of length 5 over the alphabet \{A, B, C, ..., Z\} are there?
• E.g., AZURE, BINGO, TANGO, STEVE, SARAH, ...

\[
\begin{array}{c}
\times \\
\times \\
\times \\
\times \\
\times
\end{array}
\]

How many binary string of length \(n\) over the alphabet \{0,1\}?
• E.g., 0 \(\cdots\) 0, 1 \(\cdots\) 1, 0 \(\cdots\) 01, ...

\[
\begin{array}{c}
\times \\
\times \\
\times \\
\times \\
\vdots
\end{array}
\]

\[
= 
\]

\[
= 
\]
Alice wants to buy a new laptop.

- The laptop can be blue, orange, purple, or silver.
- The SSD storage can be 128GB, 256GB, and 512GB.
- The available RAM can be 8GB or 16GB.
- The laptop comes with a 13” or with a 15” screen.

How many different laptop configurations are there?

\[ \square \times \square \times \square \times \square = \square \]
Definition. The **cartesian product** of two sets $S, T$ is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Called a 2-sequence

Order matters! $(a, b) \neq (b, a)$

\[
\begin{array}{c}
\square \times \square = |S \times T| \\
\square \times \square \times \square \times \cdots \times \square = |A_1 \times A_2 \times \cdots \times A_n|
\end{array}
\]
Definition. The power set of $S$ is the set of all subsets of $S$, \[ \{X : X \subseteq S\}. \]
Notations: $\mathcal{P}(S)$ or simply $2^S$ (which we will use).

Example. \[ 2^{\{\star, \spadesuit\}} = \{\emptyset, \{\star\}, \{\spadesuit\}, \{\star, \spadesuit\}\} \]
\[ 2^\emptyset = \{\emptyset\} \]
...

How many different subsets of $S$ are there if $|S| = n$?
Product rule example – Power set

set $S = \{e_1, e_2, e_3, \ldots, e_n\}$

subset $X = \{ \}$

$\times \times \times \times \cdots \times = \$ 

**Proposition.** $|2^S| = 2^{|S|}$
Product rule – One more example

5 books

“How many ways are there to distribute 5 books among Alice, Bob, and Charlie?”

Every book to one person, everyone gets $\geq 0$ books.
Example Book Assignment

\[ A = \{\text{book}, \text{book}^{\prime}\} \]

Alice

\[ B = \{\text{book}^{\prime}\} \]

Bob

\[ C = \{\text{book}, \text{book}^{\prime}, \text{book}^{\prime\prime}\} \]

Charlie
Book assignment – Modeling

Correct?
Poll:
A. Correct
B. Overcount
C. Undercount
D. No idea

\[ 2^5 = 32 \text{ options} \]

\[ A = \{\text{book, note}\} \]

\[ 2^5 = 32 \text{ options} \]

\[ B = \{\text{note}\} \]

\[ 2^5 = 32 \text{ options} \]

\[ C = \{\text{book, note, diary}\} \]

\[ = 32^3 \text{ assignments} \]
Problem – Overcounting

**Problem:** We are counting some invalid assignments!!! → **overcounting**!

What went wrong in the sequential process? After assigning \( A \) to Alice, \( B \) is no longer a valid option for Bob.
Book assignments – A Clever Approach
Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself. Think about counter examples to your own solution.

Food for thought: How many book assignments are there if no person can get more than 2 books?
The first concept check is out and due 9:00am before the next lecture.

The concept checks are meant to help you immediately reinforce what is learned.

Students from previous quarters have found them really useful!