Quiz Section 1

Review

1) **Sum rule.** If you can choose from EITHER one of \( n \) options, OR one of \( m \) options with NO overlap with the previous \( n \), then the number of possible outcomes of the experiment is ________________.

2) **Product rule.** In a sequential process with \( m \) steps, if there are \( n_1 \) choices for the 1st step, \( n_2 \) choices for the 2nd step (given the first choice), ..., and \( n_m \) choices for the \( m \)th step (given the previous choices), then the total number of outcomes is ________________.

3) **Permutations.** The number of ways to re-order \( n \) elements is ________.

4) **\( k \)-permutations.** The number of ways to choose a sequence of \( k \) distinct elements from a set of \( n \) elements is ________.

5) **Subsets.** The number of ways to choose a \( k \)-element subset of a set of \( n \) elements is _____.

6) **Set difference.** Is it always true that \(|A \setminus B| = |A| - |B|\)?

Task 1 – Sets

a) For each one of the following sets, give its **cardinality**, i.e., indicate how many elements it contains:
   
   - \( A = \emptyset \)
   - \( B = \{\emptyset\} \)
   - \( C = \{\emptyset\} \)
   - \( D = \emptyset, \{\emptyset\} \)

b) Let \( S = \{a, b, c\} \) and \( T = \{c, d\} \). Compute:
   
   - \( S \cup T \)
   - \( S \cap T \)
   - \( S \setminus T \)
   - \( 2^{S \setminus T} \)
   - \( S \times T \)

Task 2 – Basic Counting

a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is uniquely determined by the first 15 digits). How many credit-card numbers are there?

b) How many positive divisors does \( 1440 = 2^5 \cdot 3^2 \cdot 5 \) have?

c) How many ways are there to arrange the CSE 312 staff on a line (11 TAs, two professors) for a group picture?

d) How many ways are there to arrange the CSE 312 staff on a line so that Professors Tessaro and Beame are at the two ends of the line?
Task 3 – Seating
How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . . 
a) . . . all couples are to get adjacent seats?
b) . . . anyone can sit anywhere, except that one couple insists on not sitting in adjacent seats?

Task 4 – Weird Card Game
In how many ways can a pack of fifty-two cards (in four suits of thirteen cards each) be dealt to thirteen players, four to each, so that every player has one card from each of the suits?

Task 5 – Full Class
There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

Task 6 – HBCDEFGA
How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

Task 7 – Escape the Professor
There are 6 security professors and 7 theory professors taking part in an escape room. The solution requires that they choose 4 pairs, each consisting of one security professor and one theory professor. How many options for pairings do they have?

Task 8 – Lizards and Snakes!
Loudon has three pet lizards, Rango, a gecko named Gordon, and a goanna named Joanna, as well as two small pet snakes, Kaa and Basilisk, but only 4 terrariums to put them in. In how many different ways can he put his 5 pets in these 4 terrariums so that no terrarium has both a snake and a lizard?

Task 9 – Birthday Cake
A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

Task 10 – Photographs
Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

Task 11 – Extended Family Portrait
A group of \( n \) families, each with \( m \) members, are to be lined up for a photograph. In how many ways can the \( nm \) people be arranged if members of a family must stay together?