Quiz Section 8.5

Review

1) Maximum Likelihood Estimator (MLE): We denote the MLE of θ as $\hat{\theta}_{\text{MLE}}$ or simply $\hat{\theta}$, the parameter (or vector of parameters) that maximizes the likelihood function (probability of seeing the data).

$$\hat{\theta}_{\text{MLE}} = \arg\max_{\theta} \mathcal{L}(x_1, \ldots, x_n \mid \theta) = \arg\max_{\theta} \ln \mathcal{L}(x_1, \ldots, x_n \mid \theta)$$

2) An estimator $\hat{\theta}$ for a parameter θ of a probability distribution is **unbiased** iff $\mathbb{E}[\hat{\theta}(X_1, \ldots, X_n)] = \theta$

Task 1 – Mystery Dish!

A fancy new restaurant has opened up that features only 4 dishes. The unique feature of dining here is that they will serve you any of the four dishes randomly according to the following probability distribution: give dish A with probability 0.5, dish B with probability θ, dish C with probability 2θ, and dish D with probability $0.5 - 3\theta$. Each diner is served a dish independently. Let x_A be the number of people who received dish A, x_B the number of people who received dish B, etc, where $x_A + x_B + x_C + x_D = n$. Find the MLE for θ, $\hat{\theta}$.

Task 2 – A Red Poisson

Suppose that $x_1, \ldots, x_n$ are i.i.d. samples from a Poisson(θ) random variable, where θ is unknown. In other words, they follow the distributions $\mathbb{P}(k; \theta) = \theta^k e^{-\theta}/k!$, where $k \in \mathbb{N}$ and $\theta > 0$ is a positive real number.

Find the MLE of θ.

Task 3 – A biased estimator

In class, we showed that the maximum likelihood estimate of the variance θ_2 of a normal distribution (when both the true mean μ and true variance σ^2 are unknown) is what's called the *population variance*. That is

$$\hat{\theta}_2 = \left(\frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2) \right)$$

where $\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$ is the MLE of the mean. Is $\hat{\theta}_2$ unbiased?

Task 4 – Weather Forecast

A weather forecaster predicts sun with probability θ_1, clouds with probability $\theta_2 - \theta_1$, rain with probability $\frac{1}{2}$ and snow with probability $\frac{1}{2} - \theta_2$. This year, there have been 55 sunny days, 100 cloudy days, 160 rainy days and 50 snowy days. What is the maximum likelihood estimator for θ_1 and θ_2?