

CSE 312

Foundations of Computing II

Optional Review: Set Theory and Notation

Sets

A **set** is a collection of (distinct) elements from a **universe**.

- Order irrelevant: $\{1,2,3\} = \{3,1,2\} = \{3,2,1\} = \{1,3,2\} = \dots$
- No repetitions: $\{1,2,2,3\} = \{1,2,3\}$

Common sets

Common sets:

- Empty set
- Integers $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- Naturals $\mathbb{N} = \{0, 1, 2, 3, \dots\}$
- Reals \mathbb{R}
- Rationals $\mathbb{Q} = \left\{ \frac{a}{b} \in \mathbb{R} \mid a, b \in \mathbb{Z}, b \neq 0 \right\}$

Implicit descriptions

Often, sets are described implicitly.

$$S_1 = \{a \in \mathbb{N} \mid 1 \leq a \leq 7\}$$

What is this set? $S_1 = \{1, 2, 3, 4, 5, 6, 7\}$

$$S_2 = \{a \in \mathbb{N} \mid \exists k \in \mathbb{N}: a = 2k + 1\}$$

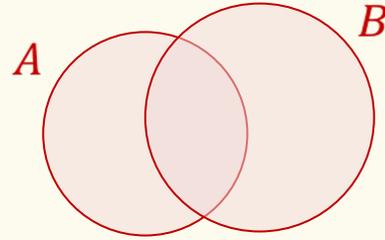
What is this set? $S_2 = \{1, 3, 5, 7, \dots\} =$ the odd naturals

unambiguous

ambiguous

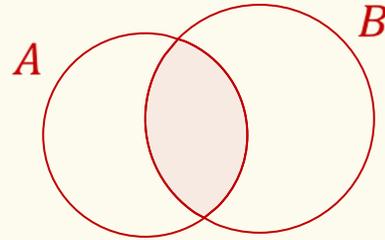
Set operations

$$A \cup B = \{x : x \in A \vee x \in B\}$$



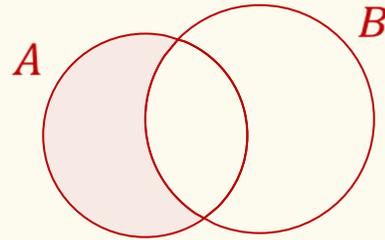
set union

$$A \cap B = \{x : x \in A \wedge x \in B\}$$



set intersection

$$A \setminus B = \{x : x \in A \wedge x \notin B\}$$



set difference

[Sometimes also: $A - B$]

Set operations (cont'd)

$$A^c = \{x : x \notin A\} = \Omega \setminus A$$

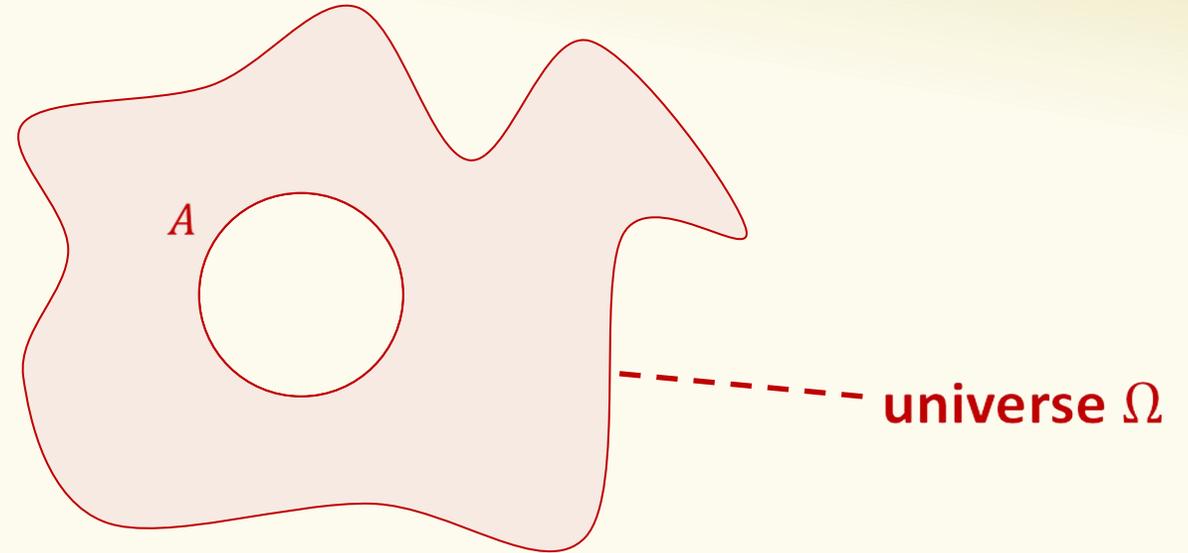
set complement

[Sometimes also: \bar{A}]

$$\text{Fact 1. } (A^c)^c = A.$$

$$\text{Fact 2. } (A \cup B)^c = A^c \cap B^c.$$

$$\text{Fact 3. } (A \cap B)^c = A^c \cup B^c.$$



“De Morgan’s Laws”

Sequences

A (finite) **sequence** (or **tuple**) is an (ordered) list of elements

- Order matters: $(1,2,3) \neq (3,2,1) \neq (1,3,2)$
- Repetitions matter: $(1,2,3) \neq (1,2,2,3) \neq (1,1,2,3)$

• **Definition.** The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Equivalent naming: 2-sequence = 2-tuple = ordered pair.

Cartesian product – cont'd

• **Definition.** The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Example.

$$\{1, 2, 3\} \times \{\star, \spadesuit\} = \{(1, \star), (2, \star), (3, \star), (1, \spadesuit), (2, \spadesuit), (3, \spadesuit)\}$$

Cartesian product – even more notation

$$S \times T \times U = \{(a, b, c) : a \in S, b \in T, c \in U\}$$

$$S \times T \times U \times V$$

...

$$\text{Notation. } S^k = \underbrace{S \times S \times \cdots \times S}_{k \text{ times}}$$