# CSE 312
# Foundations of Computing II

## Optional Review: Set Theory and Notation

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Slide Credit: Based on Stefano Tessaro's slides for 312 19au
incorporating ideas from Alex Tsun's and Anna Karlin's slides for 312 20su and 20au

# Sets

- A **set** is a collection of (distinct) elements from a **universe**.
- Order irrelevant: $\{1,2,3\} = \{3,1,2\} = \{3,2,1\} = \{1,3,2\} = \cdots$
- No repetitions: $\{1,2,2,3\} = \{1,2,3\}$

# Common sets

**Common sets:**

- **Empty set**
- **Integers** $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- **Naturals** $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
- **Reals** $\mathbb{R}$
- **Rationals** $\mathbb{Q} = \left\{ \frac{a}{b} \in \mathbb{R} \,\middle|\, a, b \in \mathbb{Z}, b \neq 0 \right\}$

# Implicit descriptions

Often, sets are described <u>implicitly</u>.

$$S_1 = \{a \in \mathbb{N} \mid 1 \leq a \leq 7\}$$

**What is this set?** $S_1 = \{1,2,3,4,5,6,7\}$

unambiguous

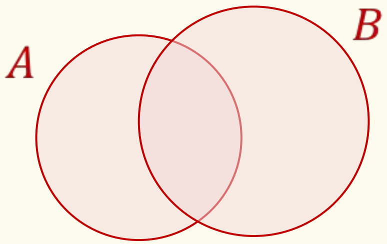
$$S_2 = \{a \in \mathbb{N} \mid \exists k \in \mathbb{N}: a = 2k + 1\}$$

**What is this set?** $S_2 = \{1,3,5,7, \ldots\}$ = the <u>odd</u> naturals

ambiguous

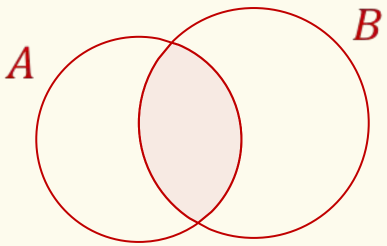
# Set operations

$$A \cup B = \{x : x \in A \vee x \in B\}$$



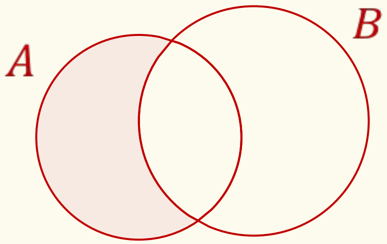

**set union**

$$A \cap B = \{x : x \in A \wedge x \in B\}$$




**set intersection**

$$A \setminus B = \{x : x \in A \wedge x \notin B\}$$




**set difference**

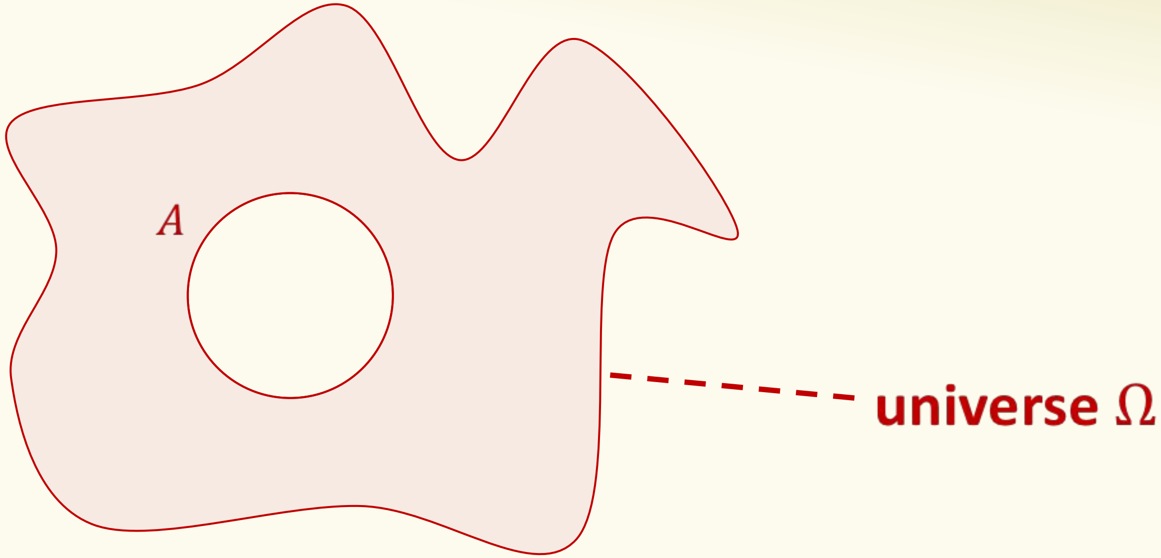
[Sometimes also: $A - B$]

# Set operations (cont'd)

$$A^c = \{x : x \notin A\} = \Omega \setminus A$$

**set complement**

[Sometimes also: $\bar{A}$]




**Fact 1.** $(A^c)^c = A$.

**Fact 2.** $(A \cup B)^c = A^c \cap B^c$.

**Fact 3.** $(A \cap B)^c = A^c \cup B^c$.

"De Morgan's Laws"

# Sequences

A (finite) **sequence** (or **tuple**) is an (ordered) list of elements

- Order matters: $(1,2,3) \neq (3,2,1) \neq (1,3,2)$
- Repetitions matter: $(1,2,3) \neq (1,2,2,3) \neq (1,1,2,3)$

**Definition.** The **cartesian product** of two sets $S, T$ is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Equivalent naming: 2-sequence = 2-tuple = ordered pair.

# Cartesian product – cont'd

- **Definition.** The **cartesian product** of two sets $S, T$ is

$$S \times T = \{(a, b): a \in S, b \in T\}$$

## Example.

$$\{1,2,3\} \times \{\bigstar, \spadesuit\} = \{(1,\bigstar), (2,\bigstar), (3,\bigstar), (1, \spadesuit), (2, \spadesuit), (3, \spadesuit)\}$$

# Cartesian product – even more notation

- $S \times T \times U = \{(a, b, c) : a \in S, b \in T, c \in U\}$

$S \times T \times U \times V$

…

**Notation.** $S^k = \underbrace{S \times S \times \cdots \times S}_{k \text{ times}}$