CSE 312

Foundations of Computing II

Lecture 1: Counting

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Rachel Lin, Hunter Schafer

Slide Credit: Based on Stefano Tessaro's slides for 312 19au
incorporating ideas from Alex Tsun's and Anna Karlin's slides for 312 20su and 20au

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Lectures and Sections (ZOOM – ZOOM - ZOOM)

- Lectures MWF
  - 9:30-10:20am or 1:30-2:20pm
  - Recorded and video released after class
  - Monday lectures are covered by Rachel, Friday lectures by Hunter
  - Wednesday lectures are covered alternatively by Rachel and Hunter
- Ask questions by writing in the chat
  - Questions will be answered periodically
  - Some questions may be deferred to the end of the lecture
  - Feel free to answer your fellow classmate's questions on chat
- Sections Thu (starts this week)
  - Not recorded, for privacy of student discussion

Questions and Discussions

- **Office hour throughout the week (starting Tuesday)**

- **Ed Discussion**

You should have received an invitation (synchronized with the class roaster)

  - Material (resources tab)
  - Announcement (discussion tab)
  - Discussion (discussion tab)

Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructors for personal issues.

Engagement

- **Checkpoints after each lecture   10%**
  - Must be done before the next lecture.
  - Simple questions to reinforce concepts taught in each class
  - Keep you engaged throughout the week, so that homework becomes less of a hurdle
- **8 Homework (Gradescope)60%**
  - Teams of 1 or 2. Submit a single solution only.
  - **Discussion outside the group must remain high-level. See examples on course webpage**
- **1 Midterm and 1 Final   15%+15%**
  - Teams of 1 or 2. Submit individual solutions.
  - **No Discussion outside the group**

Check out the syllabus for policies on late submission for check points and HW

More details see

Course Webpage
https://courses.cs.washington.edu/courses/cse312/21wi/

Foundations of Computing II

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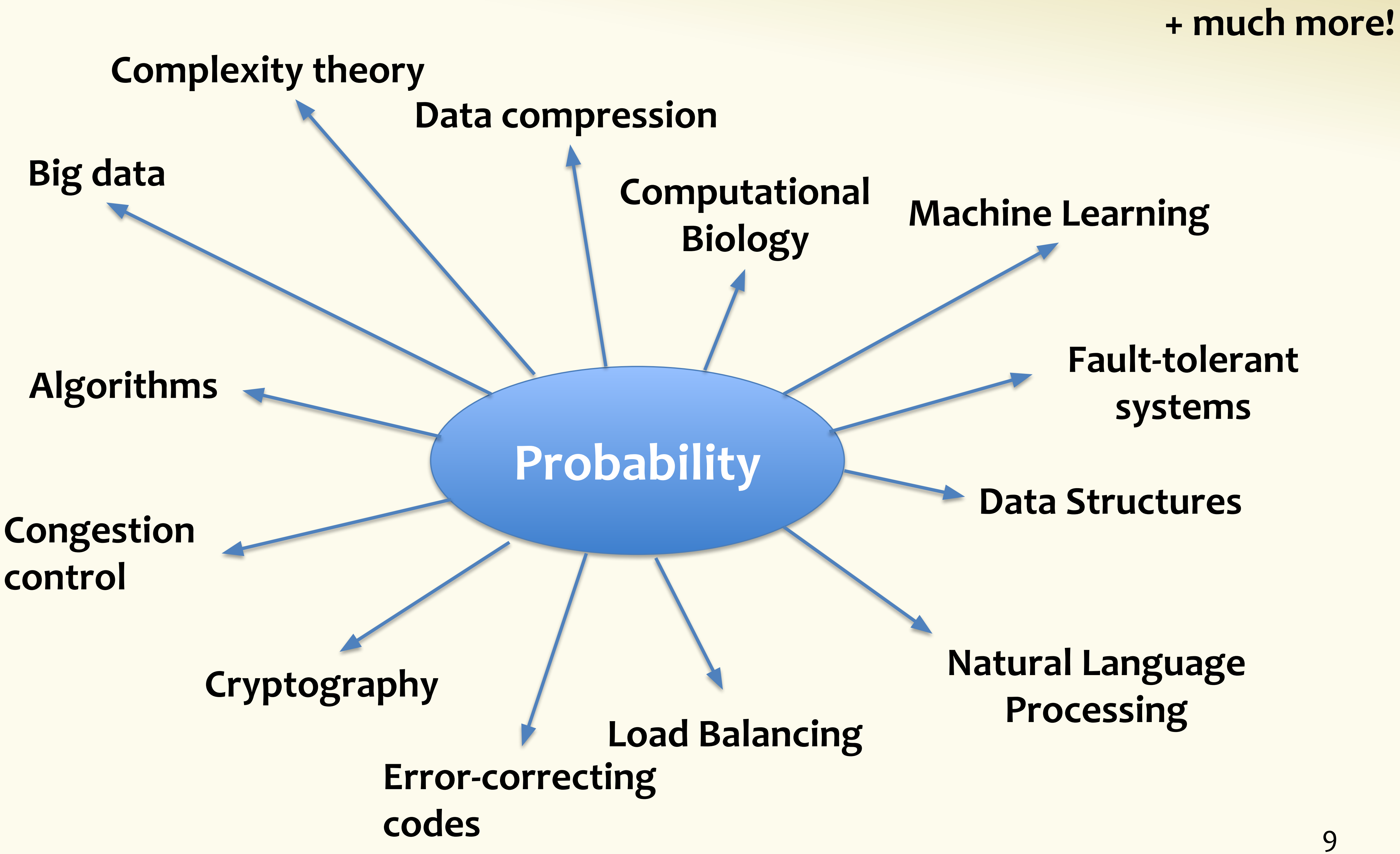
Introduction to Probability & Statistics

for computer scientists

What is probability??

Why probability?!


+ much more!
Complexity theory
Data compression
Big data
Computational Biology
Machine Learning
Algorithms
Fault-tolerant systems
Probability
Data Structures
Congestion control
Cryptography
Natural Language Processing
Load Balancing
Error-correcting codes

Content

- Counting (basis of discrete probability)
  - Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle
- What is probability
  - Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance
- Properties of probability
  - Various inequalities, Zoom of discrete random variables, Concentration, Tail bounds
- Continuous Probability
  - Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation
- Applications
  - A sample of randomized algorithms, differential privacy, learning …

Today: Counting

We are interested in counting the number of objects with a certain given property.

"How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?"

"How many integer solutions $(x, y, z) \in \mathbb{Z}^3$ does the equation $x^3 + y^3 = z^3$ have?"

Generally: Question boils down to computing cardinality $|S|$ of some given set S.

(Discrete) Probability and Counting are Twin Brothers

"What is the probability that a random student from CSE312 has black hair?"

$$= \frac{\#\ students\ with\ black\ hair}{\#students}$$

Sum Rule

If you can choose from

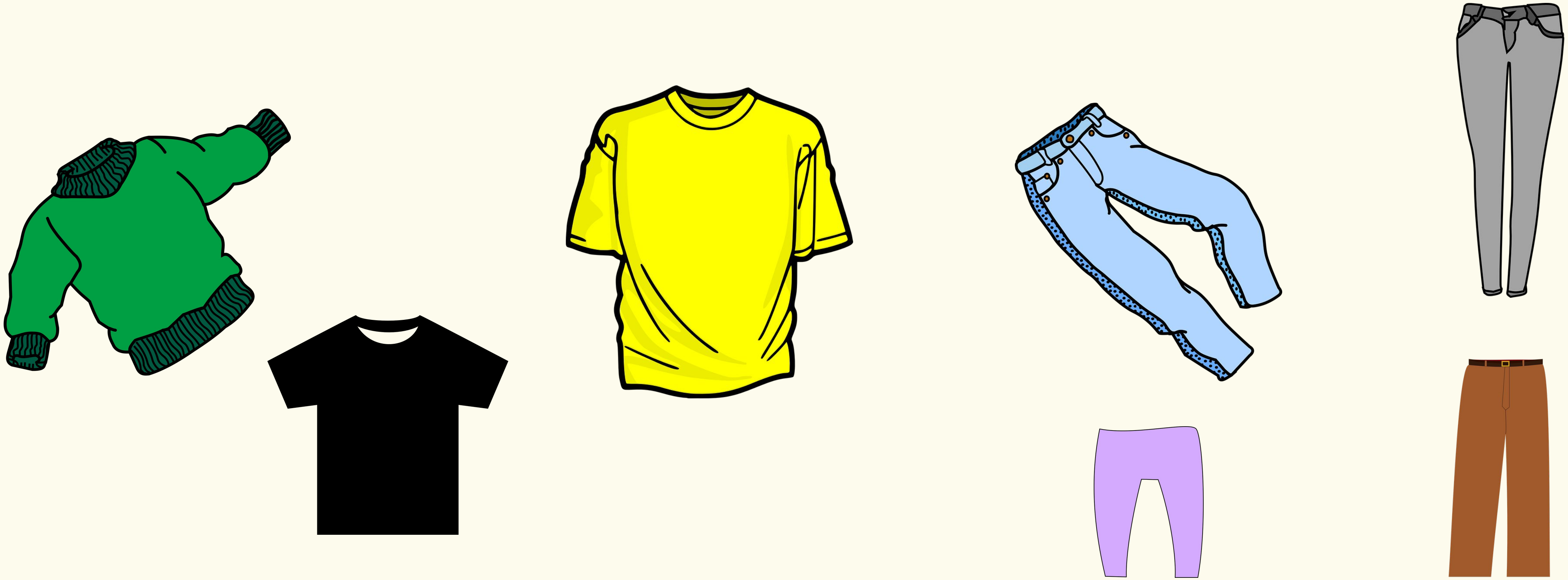
- Either one of n options,
- OR one of m options with NO overlap with the previous n,

then the number of possible outcomes of the experiment is

$$n + m$$

Counting “outfits”

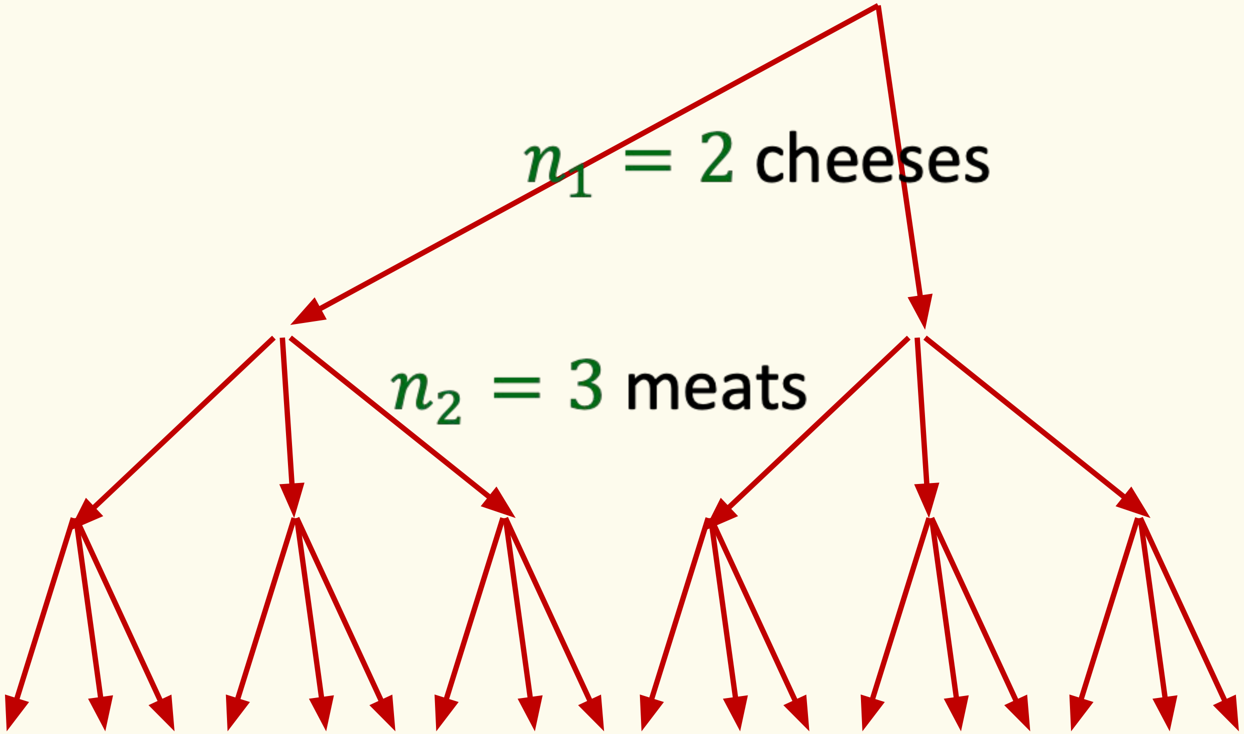
If an outfit consists of **either** a top **or** a bottom, how many outfits are possible?



Product Rule: In a sequential process, there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), … , and
- n_m choices for the m^{th} step (given the previous choices),

then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$



Example: "How many subways?"

x x =

$n_3 = 3$ veggies

Example – Strings

- How many string of length 5 over the alphabet $\{A, B, C, \dots, Z\}$ are there?
  - E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

□ × □ × □ × □ × □ = □

How many binary string of length n over the alphabet $\{0,1\}$?

- E.g., $0 \cdots 0, 1 \cdots 1, 0 \cdots 01, \dots$

□ × □ × □ × … × □ = □

Example – Laptop customization

Alice wants to buy a new laptop.

- The laptop can be **blue**, **orange**, **purple**, or **silver**.
- The SSD storage can be **128GB**, **256GB**, and **512GB**
- The available RAM can be **8GB** or **16GB.**
- The laptop comes with a **13”** or with a **15”** screen.

How many different laptop configurations are there?

	x		x		x		=	

Example -- Cartesian Product

- **Definition.** The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Called a 2-sequence
Order matters! $(a, b) \neq (b, a)$

$$\square \times \square = |S \times T|$$

$$\square \times \square \times \square \times \ldots \times \square = |A_1 \times A_2 \times \cdots \times A_n|$$

Example – Power set

Definition. The **power set** of S is

$$2^S \stackrel{\text{def}}{=} \{X : X \subseteq S\}$$

Example. $2^{\{\bigstar,\spadesuit\}} = \{\emptyset, \{\bigstar\}, \{\spadesuit\}, \{\bigstar, \spadesuit\}\}$

$2^{\emptyset} = \{\emptyset\}$

…

How many different subsets of S are there? That is $|2^S|$?

Proposition. $|2^S| = 2^{|S|}$

How to design a sequential process that produces a subset?

Example – Power set

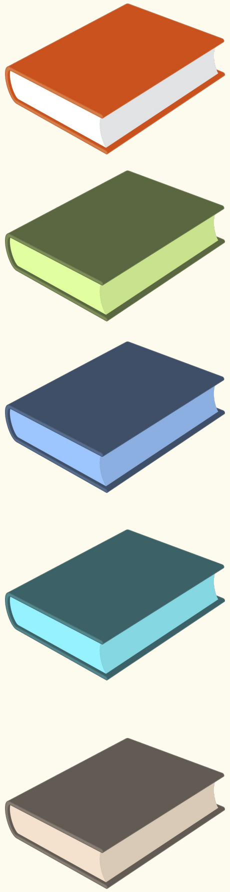
$$S = \{e_1, e_2, e_3, \cdots, e_n\}$$

$$X = \{ \qquad\qquad\qquad \}$$

$$\square \times \square \times \square \times \ldots \times \square = \square$$

Product rule – One more example

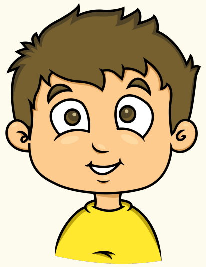
5 books



"How many ways are there to distribute 5 books among Alice, Bob, and Charlie?"

Every book to one person, everyone gets ≥ 0 books.

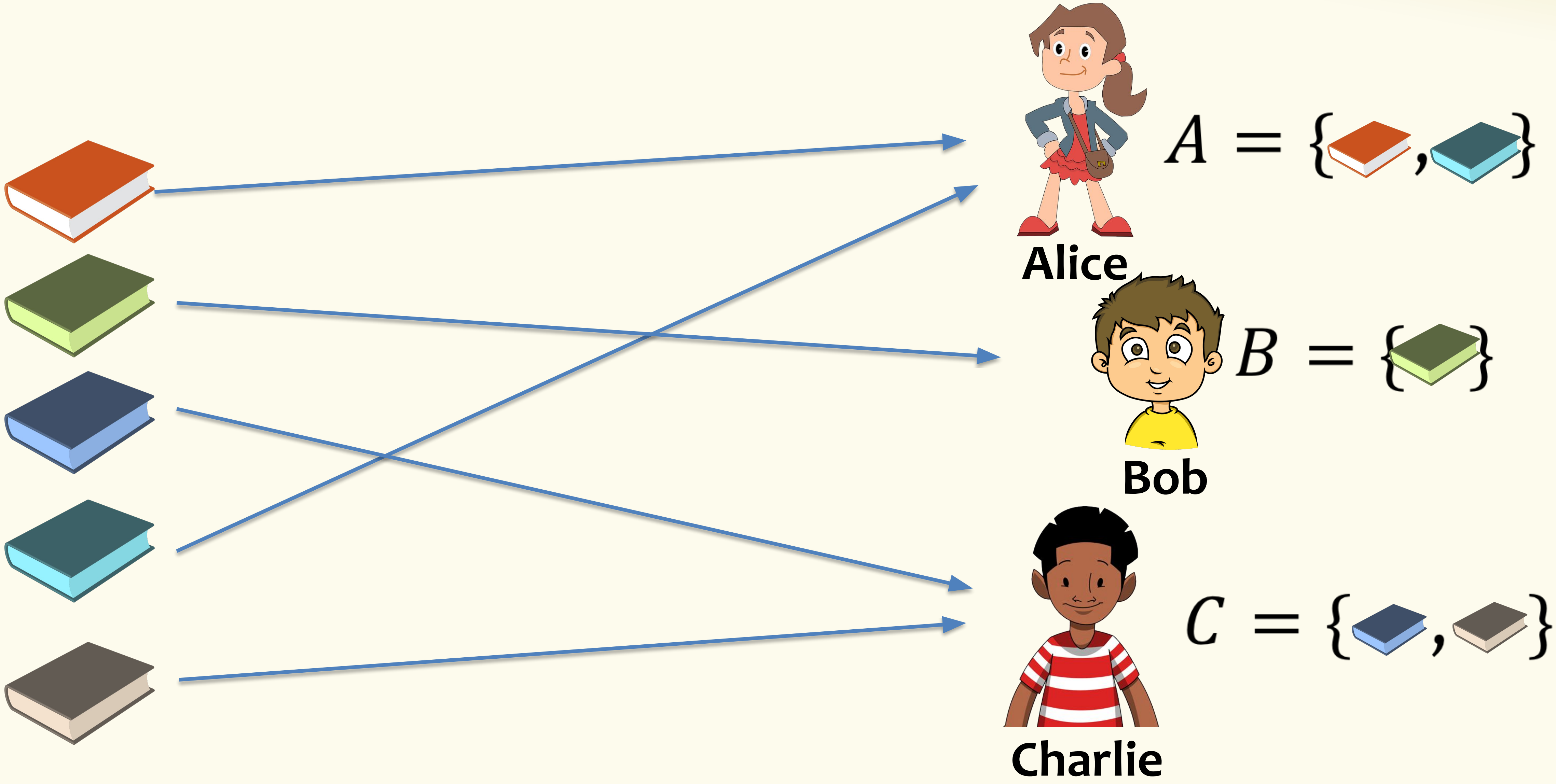
Alice



Bob

Charlie

Example Book Assignment



Book assignment – Modeling

Correct?

Poll:

A. right

B. Overcount

C. Undercount

D. No idea

https://pollev.com/rachel312

$2^5 = 32$ options

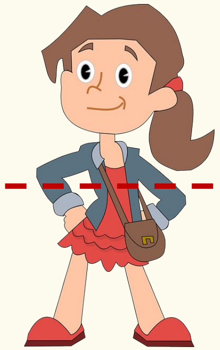
×

$2^5 = 32$ options

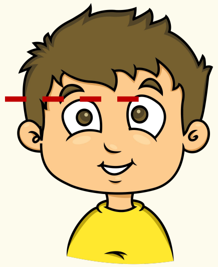
×

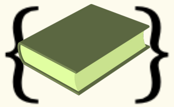
$2^5 = 32$ options

= 32^3 assignment



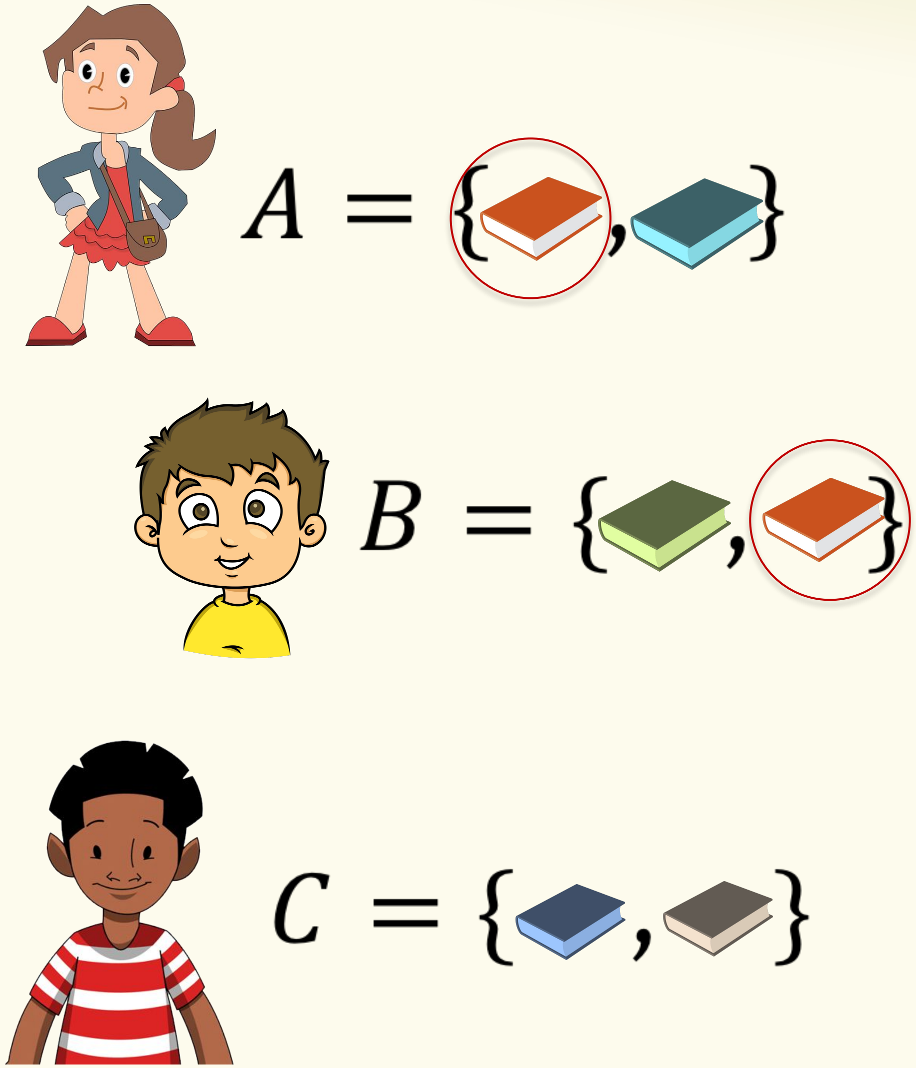
$A = \{$  $\}$



$B = \{$  $\}$

$C = \{$  $\}$

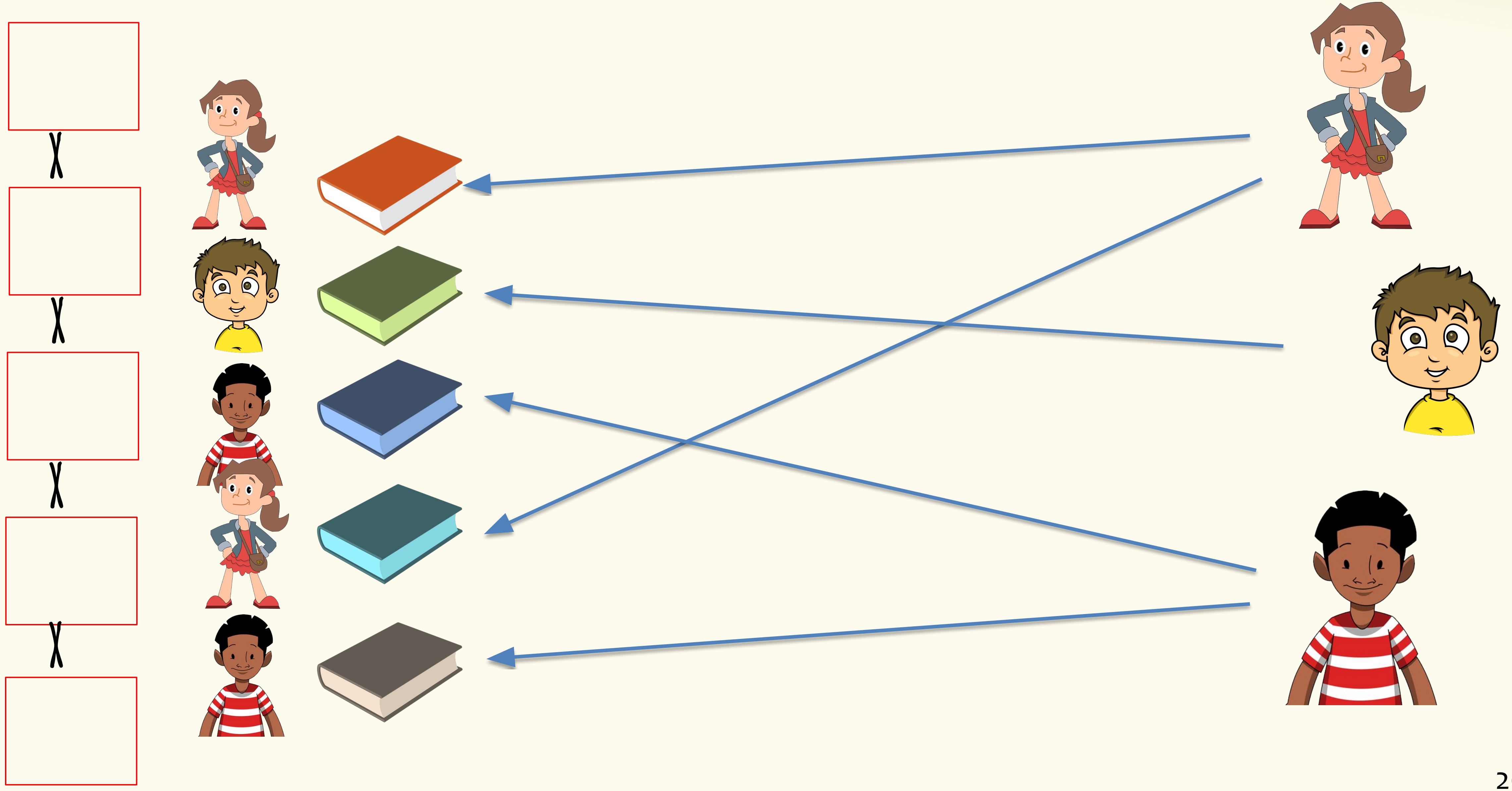
Problem – Overcounting



Problem: We are counting some <u>invalid</u> assignments!!!
$\rightarrow$ <u>**overcounting**</u>!

What went wrong in the sequential process?
After assigning A to Alice,
B is no longer a valid option for Bob

Book assignments – A Clever Approach



Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself
Think about counter examples to your own solution.

Food for thought: How many book assignment are there if no person can get more than 2 books?

The first concept check is out and due 9:00am before the next lecture

The concept checks are meant to help you immediately reinforce what is learned.

Students from the last quarter found them really useful!