CSE 312
Foundations of Computing II

Lecture 1: Counting

Slide Credit: Based on Stefano Tessaro’s slides for 312 19au incorporating ideas from Alex Tsun’s and Anna Karlin’s slides for 312 20su and 20au
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Lectures and Sections (ZOOM – ZOOM - ZOOM)

• Lectures MWF
  – 9:30-10:20am or 1:30-2:20pm
  – Recorded and video released after class
  – Monday lectures are covered by Rachel, Friday lectures by Hunter
  – Wednesday lectures are covered alternatively by Rachel and Hunter

• Ask questions by writing in the chat
  – Questions will be answered periodically
  – Some questions may be deferred to the end of the lecture
  – Feel free to answer your fellow classmate’s questions on chat

• Sections Thu (starts this week)
  – Not recorded, for privacy of student discussion
Questions and Discussions

• **Office hour throughout the week (starting Tuesday)**

• **Ed Discussion**
  You should have received an invitation (synchronized with the class roaster)
  – Material (resources tab)
  – Announcement (discussion tab)
  – Discussion (discussion tab)

Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructors for personal issues.
Engagement

• **Checkpoints after each lecture  10%**
  – Must be done before the next lecture.
  – Simple questions to reinforce concepts taught in each class
  – Keep you engaged throughout the week, so that homework becomes less of a hurdle

• **8 Homework (Gradescope) 60%**
  – Teams of 1 or 2. Submit a single solution only.
  – Discussion outside the group must remain high-level. See examples on course webpage

• **1 Midterm and 1 Final  15%+15%**
  – Teams of 1 or 2. Submit individual solutions.
  – No Discussion outside the group

Check out the syllabus for policies on late submission for check points and HW
More details see

Course Webpage

https://courses.cs.washington.edu/courses/cse312/21wi/
Foundations of Computing II

= Introduction to Probability & Statistics for computer scientists

What is probability??
Why probability?!
Probability

- Complexity theory
- Data compression
- Computational Biology
- Machine Learning
- Fault-tolerant systems
- Data Structures
- Natural Language Processing
- Load Balancing
- Error-correcting codes
- Cryptography
- Algorithms
- Congestion control
- Big data

+ much more!
Content

• Counting (basis of discrete probability)
  – Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle

• What is probability
  – Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance

• Properties of probability
  – Various inequalities, Zoom of discrete random variables, Concentration, Tail bounds

• Continuous Probability
  – Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation

• Applications
  – A sample of randomized algorithms, differential privacy, learning ...
Today: Counting
We are interested in counting the number of objects with a certain given property.

“How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?”

“How many integer solutions \((x, y, z) \in \mathbb{Z}^3\) does the equation \(x^3 + y^3 = z^3\) have?”

Generally: Question boils down to computing cardinality \(|S|\) of some given set \(S\).
"What is the probability that a random student from CSE312 has black hair?"

$$\frac{\# \text{ students with black hair}}{\# \text{ students}}$$
Sum Rule

If you can choose from

- "Either" one of $n$ options,
- "OR" one of $m$ options with NO overlap with the previous $n$,

then the number of possible outcomes of the experiment is

$$n + m$$
Counting “outfits”

If an outfit consists of either a top or a bottom, how many outfits are possible?
**Product Rule:** In a sequential process, there are
- $n_1$ choices for the first step,
- $n_2$ choices for the second step (given the first choice), …, and
- $n_m$ choices for the $m^{th}$ step (given the previous choices),
then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$

**Example:** “How many subways?"

$n_1 = 2$ cheeses

$n_2 = 3$ meats

$n_3 = 3$ veggies
Example – Strings

How many string of length 5 over the alphabet \{A, B, C, ..., Z\} are there?

• E.g., AZURE, BINGO, TANGO, STEVE, SARAH, ...

\[
\begin{array}{cccccc}
\times & \times & \times & \times & \times & = \\
\end{array}
\]

How many binary string of length \(n\) over the alphabet \{0,1\}?

• E.g., 0 \cdots 0, 1 \cdots 1, 0 \cdots 01, ...

\[
\begin{array}{cccccc}
\times & \times & \times & \times & \times & = \\
\end{array}
\]
Example – Laptop customization

Alice wants to buy a new laptop.
• The laptop can be blue, orange, purple, or silver.
• The SSD storage can be 128GB, 256GB, and 512GB.
• The available RAM can be 8GB or 16GB.
• The laptop comes with a 13” or with a 15” screen.

How many different laptop configurations are there?

\[
\text{Number of configurations} = \text{Color} \times \text{Storage} \times \text{RAM} \times \text{Screen Size}
\]
**Definition.** The **cartesian product** of two sets $S, T$ is

$$S \times T = \{(a, b): a \in S, b \in T\}$$

Called a 2-sequence
Order matters! $(a, b) \neq (b, a)$

$$\begin{array}{c}
\times \\
\times \\
\times \\
\times ...	imes \\
\times \\
\end{array} = |S \times T|$$

$$\begin{array}{c}
\times \\
\times \\
\times \\
\times ...	imes \\
\times \\
\end{array} = |A_1 \times A_2 \times \cdots \times A_n|$$
Example – Power set

**Definition.** The **power set** of $S$ is

$$2^S \overset{\text{def}}{=} \{ X : X \subseteq S \}$$

Example.  

$$2^{\{\star, \spadesuit\}} = \{ \emptyset, \{\star\}, \{\spadesuit\}, \{\star, \spadesuit\} \}$$

$$2^\emptyset = \{ \emptyset \}$$

... 

How many different subsets of $S$ are there? That is $|2^S|$?

**Proposition.** $|2^S| = 2^{|S|}$

How to design a sequential process that produces a subset?
Example – Power set

\[ S = \{e_1, e_2, e_3, \ldots, e_n\} \]

\[ X = \{ \} \]

\[ \begin{array}{ccccccc}
X & X & X & X & \ldots & X & = \\
\end{array} \]
Product rule – One more example

5 books

“How many ways are there to distribute 5 books among Alice, Bob, and Charlie?”

Every book to one person, everyone gets $\geq 0$ books.
Example Book Assignment

\[ A = \{\text{book}, \text{book}\} \]

Alice

\[ B = \{\text{book}\} \]

Bob

\[ C = \{\text{book}, \text{book}\} \]

Charlie
Book assignment – Modeling

Correct?

Poll:
A. right
B. Overcount
C. Undercount
D. No idea

\[ 2^5 = 32 \text{ options} \]

\[ A = \{ \text{book, book} \} \]

\[ B = \{ \text{book} \} \]

\[ C = \{ \text{book, book, book} \} \]

\[ = 32^3 \text{ assignment} \]

https://pollev.com/rachel312
Problem – Overcounting

**Problem:** We are counting some **invalid** assignments!!!
→ **overcounting**!

What went wrong in the sequential process?
After assigning $A$ to Alice, $B$ is no longer a valid option for Bob
Book assignments – A Clever Approach
Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself. Think about counter examples to your own solution.

Food for thought: How many book assignments are there if no person can get more than 2 books?
The first concept check is out and due 9:00am before the next lecture.

The concept checks are meant to help you immediately reinforce what is learned.

Students from the last quarter found them really useful!