CSE 312

Foundations of Computing II

Lecture 1: Counting

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Rachel Lin, Hunter Schafer

Slide Credit: Based on Stefano Tessaro's slides for 312 19au
incorporating ideas from Alex Tsun's and Anna Karlin's slides for 312 20su and 20au

Instructors

Hunter Schafer

hschafer@cs

Assistant Teaching Professor

https://homes.cs.washington.edu/~hschafer/

Huijia (Rachel) Lin

rachel@cs

Associate Professor

Specialty: **Cryptography**

https://homes.cs.washington.edu/~rachel/

A Team of fantastic TAs

JOSEPH AZIZEH
ASHWIN KISHIN BANWARI
Harrison Bay
EGE CAGLAR
RAHUL CHANDRA
XINYUE CHEN
Anna Dmitrievna Goncharenko
William Howard-Snyder
SHREYA JAYARAMAN
ALEKS JOVCIC
Logan Milandin
TYLER PHUC BAO NGUYEN
Jerome Paliakkara
RISHABH SURESH PATTED
PHAWIN PRONGPAOPHAN
Zoey Shi
LUXI WANG
YUKAI YAN

Lectures and Sections (ZOOM – ZOOM - ZOOM)

- **Lectures MWF**
  - 9:30-10:20am or 1:30-2:20pm
  - Recorded and video released after class
  - Monday lectures are covered by Rachel, Friday lectures by Hunter
  - Wednesday lectures are covered alternatively by Rachel and Hunter
- **Ask questions by writing in the chat**
  - Questions will be answered periodically
  - Some questions may be deferred to the end of the lecture
  - Feel free to answer your fellow classmate's questions on chat
- **Sections Thu (starts this week)**
  - Not recorded, for privacy of student discussion

Questions and Discussions

- **Office hour throughout the week (starting Tuesday)**

- **Ed Discussion**

You should have received an invitation (synchronized with the class roaster)

  - Material (resources tab)
  - Announcement (discussion tab)
  - Discussion (discussion tab)

Use Ed discussion forum as much as possible. You can make private posts that only the staff can view! Email instructors for personal issues.

Engagement

- **Checkpoints after each lecture 10%**
  - Must be done before the next lecture.
  - Simple questions to reinforce concepts taught in each class
  - Keep you engaged throughout the week, so that homework becomes less of a hurdle
- **8 Homework (Gradescope) 60%**
  - Teams of 1 or 2. Submit a single solution only.
  - **Discussion outside the group must remain high-level. See examples on course webpage**
- **1 Midterm and 1 Final 15%+15%**
  - Teams of 1 or 2. Submit individual solutions.
  - **No Discussion outside the group**

Check out the syllabus for policies on late submission for check points and HW

More details see

Course Webpage

https://courses.cs.washington.edu/courses/cse312/21wi/

Foundations of Computing II

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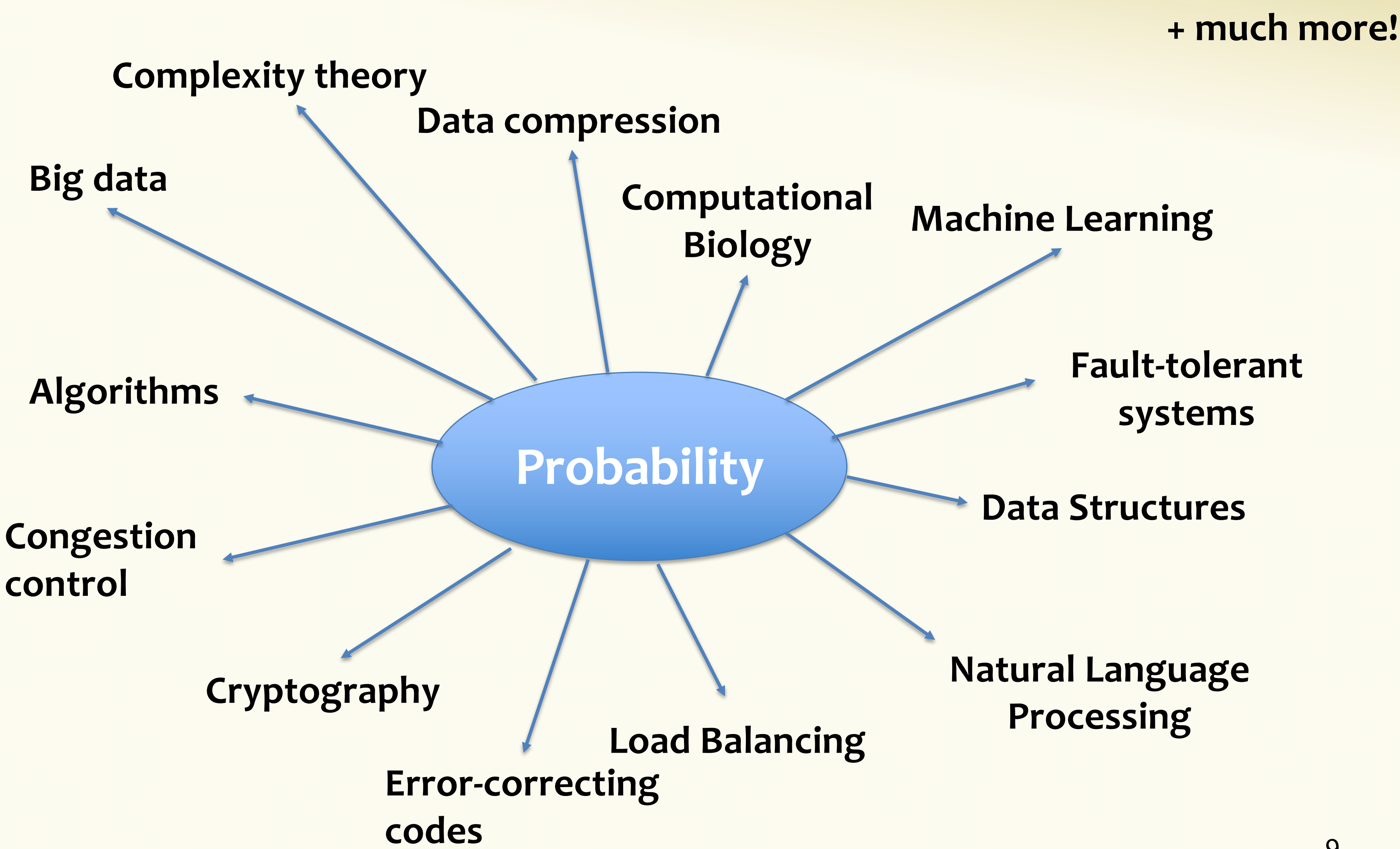
Introduction to Probability & Statistics

for computer scientists

What is probability??

Why probability?!


+ much more!
Complexity theory
Data compression
Big data
Computational Biology
Machine Learning
Algorithms
Fault-tolerant systems
Probability
Data Structures
Congestion control
Cryptography
Natural Language Processing
Load Balancing
Error-correcting codes

Content

- Counting (basis of discrete probability)
  - Counting, Permutation, Combination, inclusion-exclusion, Pigeonhole Principle
- What is probability
  - Probability space, events, basic properties of probabilities, conditional probability, independence, expectation, variance
- Properties of probability
  - Various inequalities, Zoom of discrete random variables, Concentration, Tail bounds
- Continuous Probability
  - Probability Density Functions, Cumulative Density Functions, Uniform, Exponential, Normal distributions, Central Limit Theorem, Estimation
- Applications
  - A sample of randomized algorithms, differential privacy, learning …

Today: Counting

We are interested in counting the number of objects with a certain given property.

"How many ways are there to assign 7 TAs to 5 sections, such that each section is assigned to two TAs, and no TA is assigned to more than two sections?"

"How many integer solutions $(x, y, z) \in \mathbb{Z}^3$ *does the equation* $x^3 + y^3 = z^3$ *have?"*

Generally: Question boils down to computing cardinality $|S|$ of some given set S.

(Discrete) Probability and Counting are Twin Brothers

"What is the probability that a random student from CSE312 has black hair?"

$$= \frac{\#\, students\, with\, black\, hair}{\#students}$$

Sum Rule

If you can choose from

- Either one of n options,
- OR one of m options with NO overlap with the previous n,

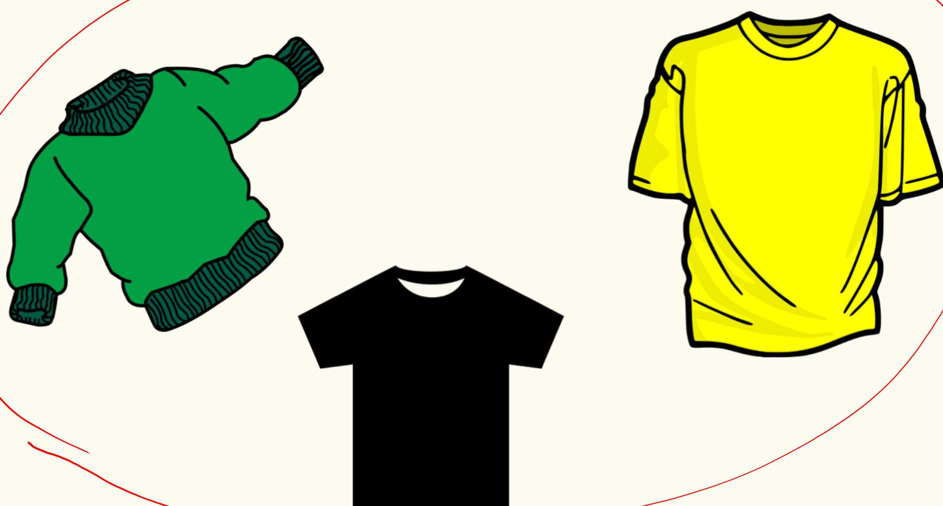
then the number of possible outcomes of the experiment is

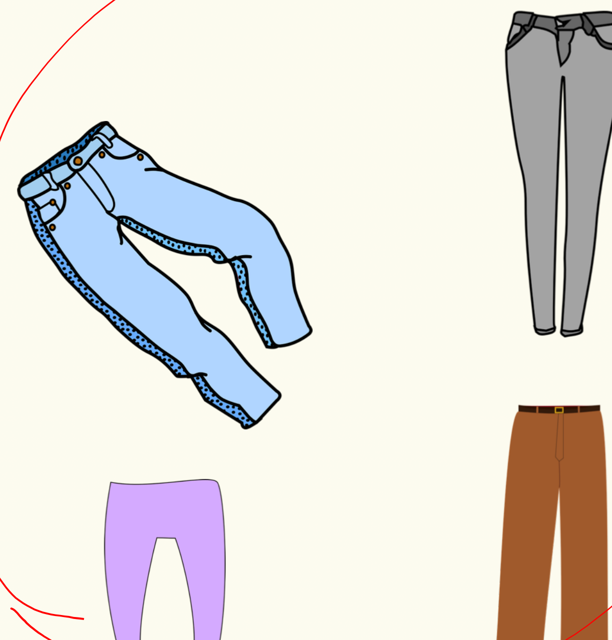
$$n + m$$

Counting "outfits"

If an outfit consists of **either** a top **or** a bottom, how many outfits are possible?

$3 + 4 = 7$





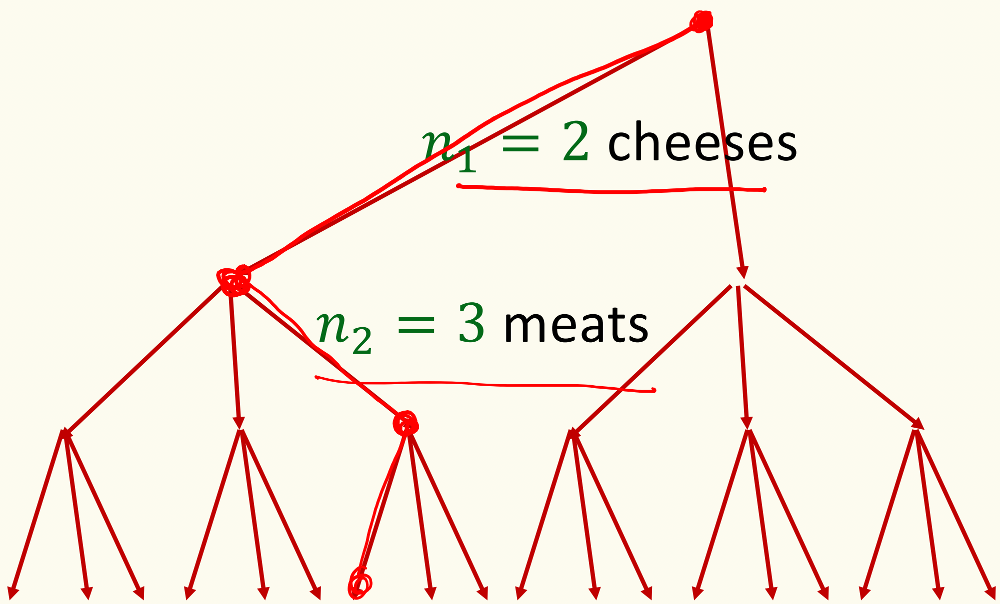
Product Rule: In a sequential process, there are

- n_1 choices for the first step,
- n_2 choices for the second step (given the first choice), …, and
- n_m choices for the m^{th} step (given the previous choices),

then the total number of outcomes is $n_1 \times n_2 \times \cdots \times n_m$

Example: "How many subways?"

$\boxed{2}$ **x** $\boxed{3}$ **x** $\boxed{3}$ **=** $\boxed{18}$



$n_3 = 3$ veggies

Example – Strings

How many string of length 5 over the alphabet $\{A, B, C, \ldots, Z\}$ are there?

- E.g., AZURE, BINGO, TANGO, STEVE, SARAH, …

$\boxed{26}$ x $\boxed{26}$ x $\boxed{26}$ x $\boxed{26}$ x $\boxed{26}$ = $\boxed{26^5}$

How many binary string of length n over the alphabet $\{0,1\}$?

- E.g., $0 \cdots 0$, $1 \cdots 1$, $0 \cdots 01$, …

$\underbrace{\boxed{2} \text{ x } \boxed{2} \text{ x } \boxed{2} \text{ x} \ldots \text{x } \boxed{2}}_{n}$ = $\boxed{2^n}$

Example – Laptop customization

Alice wants to buy a new laptop.

- The laptop can be **blue**, **orange**, **purple**, or **silver**.
- The SSD storage can be **128GB**, **256GB**, and **512GB**
- The available RAM can be **8GB** or **16GB.**
- The laptop comes with a **13”** or with a **15”** screen.

How many different laptop configurations are there?

$\boxed{4}$ **x** $\boxed{3}$ **x** $\boxed{2}$ **x** $\boxed{2}$ **=** $\boxed{48}$

Example -- Cartesian Product

Definition. The **cartesian product** of two sets S, T is

$$S \times T = \{(a, b) : a \in S, b \in T\}$$

Called a 2-sequence
Order matters! $(a, b) \neq (b, a)$

$\boxed{|S|}$ **x** $\boxed{|T|}$ **=** $|S \times T|$

$\underset{e_1}{\boxed{|A_1|}}$ **x** $\underset{e_2}{\boxed{|A_2|}}$ **x** $\boxed{|A_3|}$ **x … x** $\underset{e_n}{\boxed{|A_n|}}$ **=** $|A_1 \times A_2 \times \cdots \times A_n|$

Example – Power set

$=$

Definition. The **power set** of S is

$$2^S \overset{\text{def}}{=} \{X : X \subseteq S\}$$

Example. $2^{\{\bigstar, \spadesuit\}} = \{\emptyset, \{\bigstar\}, \{\spadesuit\}, \{\bigstar, \spadesuit\}\} \longrightarrow 2^2 = 4$

$2^{\emptyset} = \{\emptyset\} \longrightarrow 2^0 = 1$

…

How many different subsets of S are there? That is $|2^S|$?

Proposition. $|2^S| = 2^{|S|}$

$|2^{\emptyset}| = 2^{|\emptyset|} = 2^0 = 1$

How to design a sequential process that produces a subset?

Example – Power set

$$S = \{e_1, e_2, e_3, \cdots, e_n\} \quad n = |S|$$

$$\cup$$

include / not include

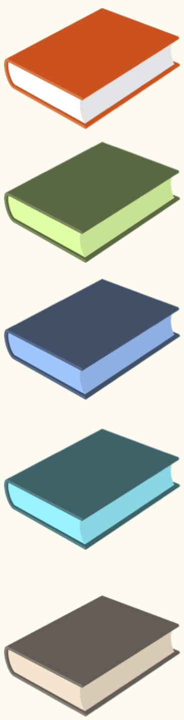
$$X = \{ e_1, \_\_, e_3, \ldots, e_n \}$$

$$1, 0, 1, \ldots, 1$$

$$\underbrace{\boxed{2} \times \boxed{2} \times \boxed{2} \times \ldots \times \boxed{2}}_{n} = \boxed{2^n} = 2^{|S|}$$

Product rule – One more example

5 books



> ***"How many ways are there to distribute 5 books among Alice, Bob, and Charlie?"***

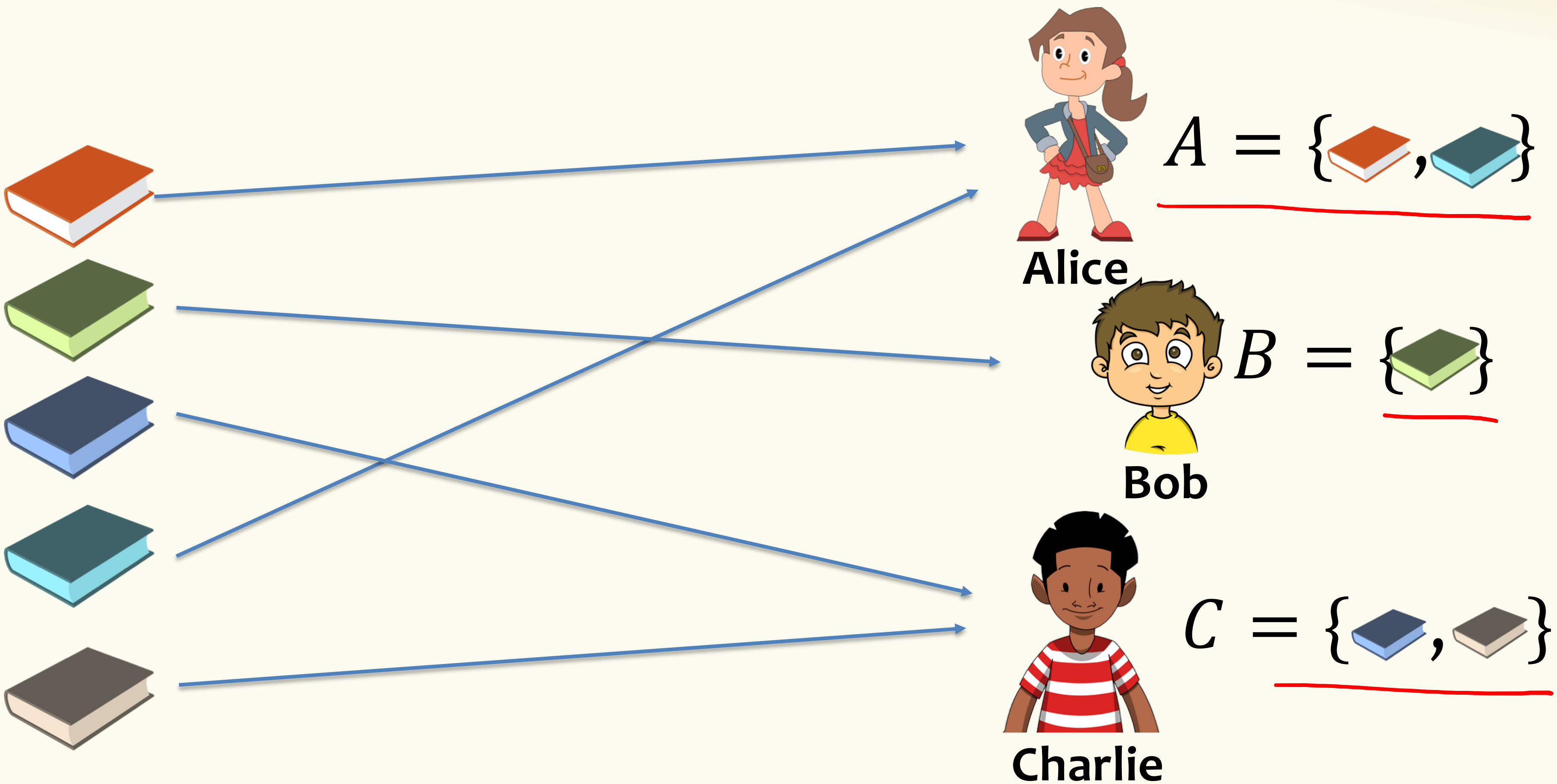
Every book to one person, everyone gets ≥ 0 books.

Alice

Bob

Charlie

Example Book Assignment



Book assignment – Modeling

Correct?

<u>Poll:</u>

A. right
B. Overcount
C. Undercount
D. No idea

https://pollev.com/rachel312

$\boxed{2^5 = 32 \text{ options}}$

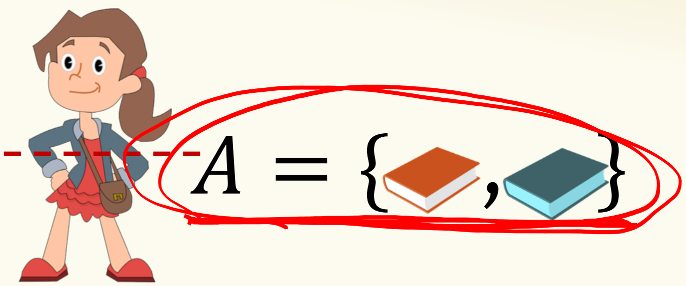
x

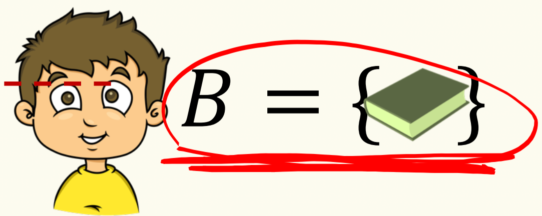
$\boxed{2^5 = 32 \text{ options}}$

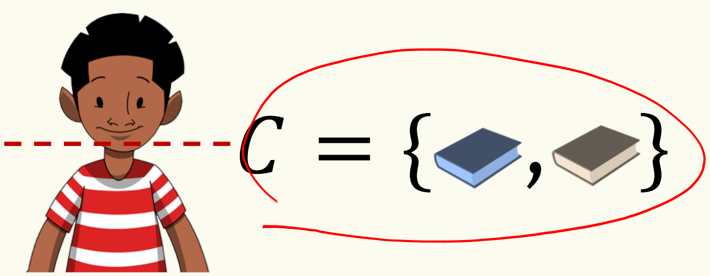
x

$\boxed{2^5 = 32 \text{ options}}$

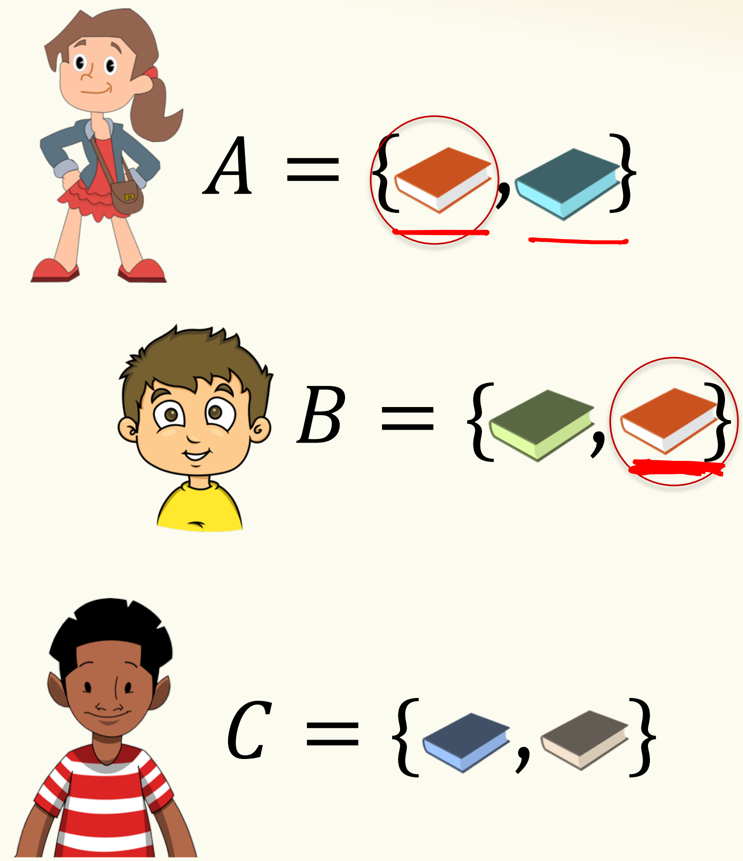
= $\boxed{32^3 \text{ assignment}}$







Problem – Overcounting

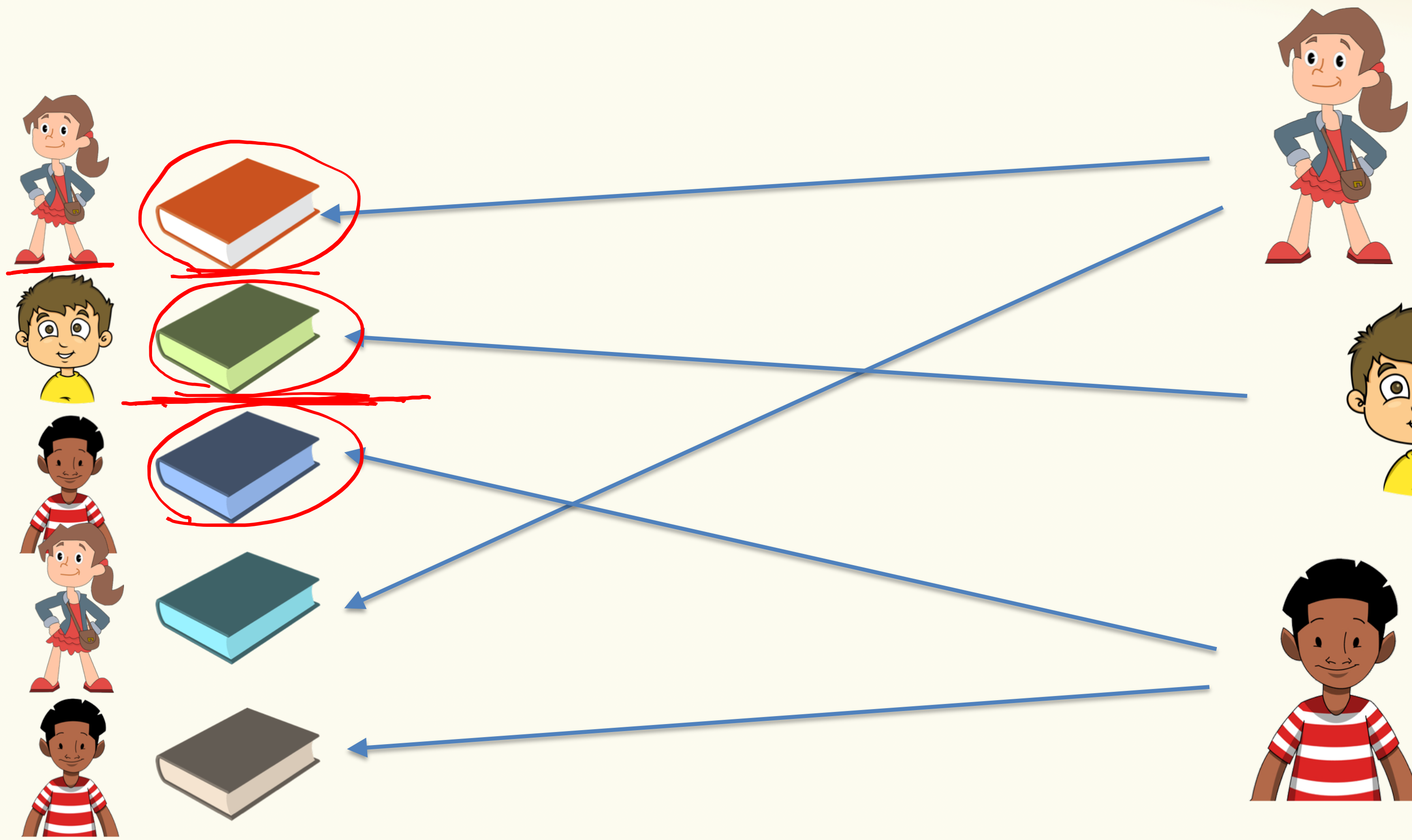


Problem: We are counting some invalid assignments!!!
$\longrightarrow$ **overcounting!**

What went wrong in the sequential process?
After assigning A to Alice,
B is no longer a valid option for Bob

Book assignments – A Clever Approach

$\boxed{3} \times \boxed{3} \times \boxed{3} \times \boxed{3} \times \boxed{3} = 3^5$



Lesson: Representation of what we are counting is very important!

Tip: Use different methods to double check yourself
Think about counter examples to your own solution.

Food for thought: How many book assignment are there if no person can get more than 2 books?

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The first concept check is out and due 9:00am before the next lecture

The concept checks are meant to help you immediately reinforce what is learned.

Students from the last quarter found them really useful!