CSE 312

Foundations of Computing II

Lecture 28: Differential Privacy

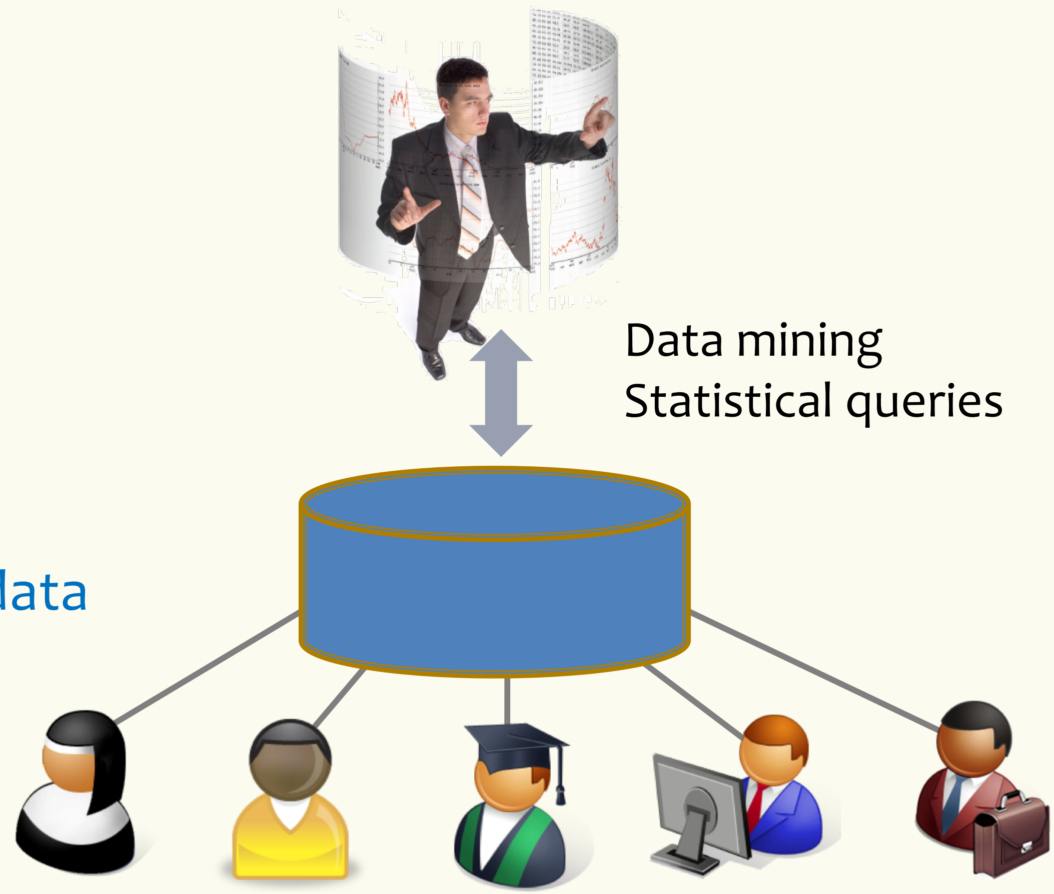
PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Rachel Lin, Hunter Schafer

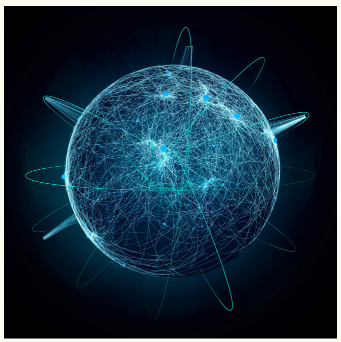
Slide Credit: Based on Stefano Tessaro's slides for 312 19au
incorporating ideas from Alex Tsun's and Anna Karlin's slides for 312 20su and 20au

Setting

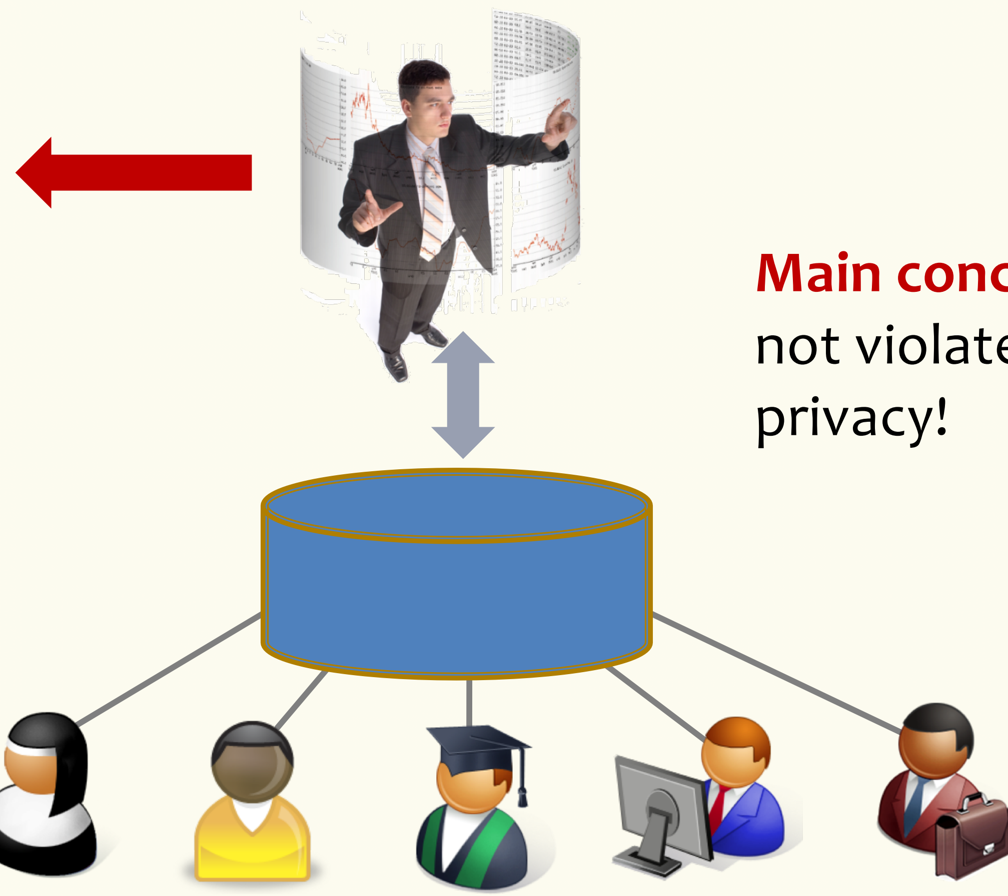
Medical data
Query logs
Social network data
...



Setting – Data Release



Internet



Publish:
Aggregated data, e.g., outcome of medical study, research paper, …

Main concern: Do not violate user privacy!

Example – Linkage Attack

[Sweeney '00]

- The Commonwealth of Massachusetts Group Insurance Commission (GIC) releases 135,000 records of patient encounters, each with 100 attributes
  - Relevant attributes removed, but ZIP, birth date, gender available
  - Considered "safe" practice
- Public voter registration record
  - Contain, among others, name, address, ZIP, birth date, gender

"Linkage"

- Allowed identification of medical records of William Weld, governor of MA at that time
  - He was the only man in his zip code with his birth date …

+More attacks! (cf. Netflix grand prize challenge!)

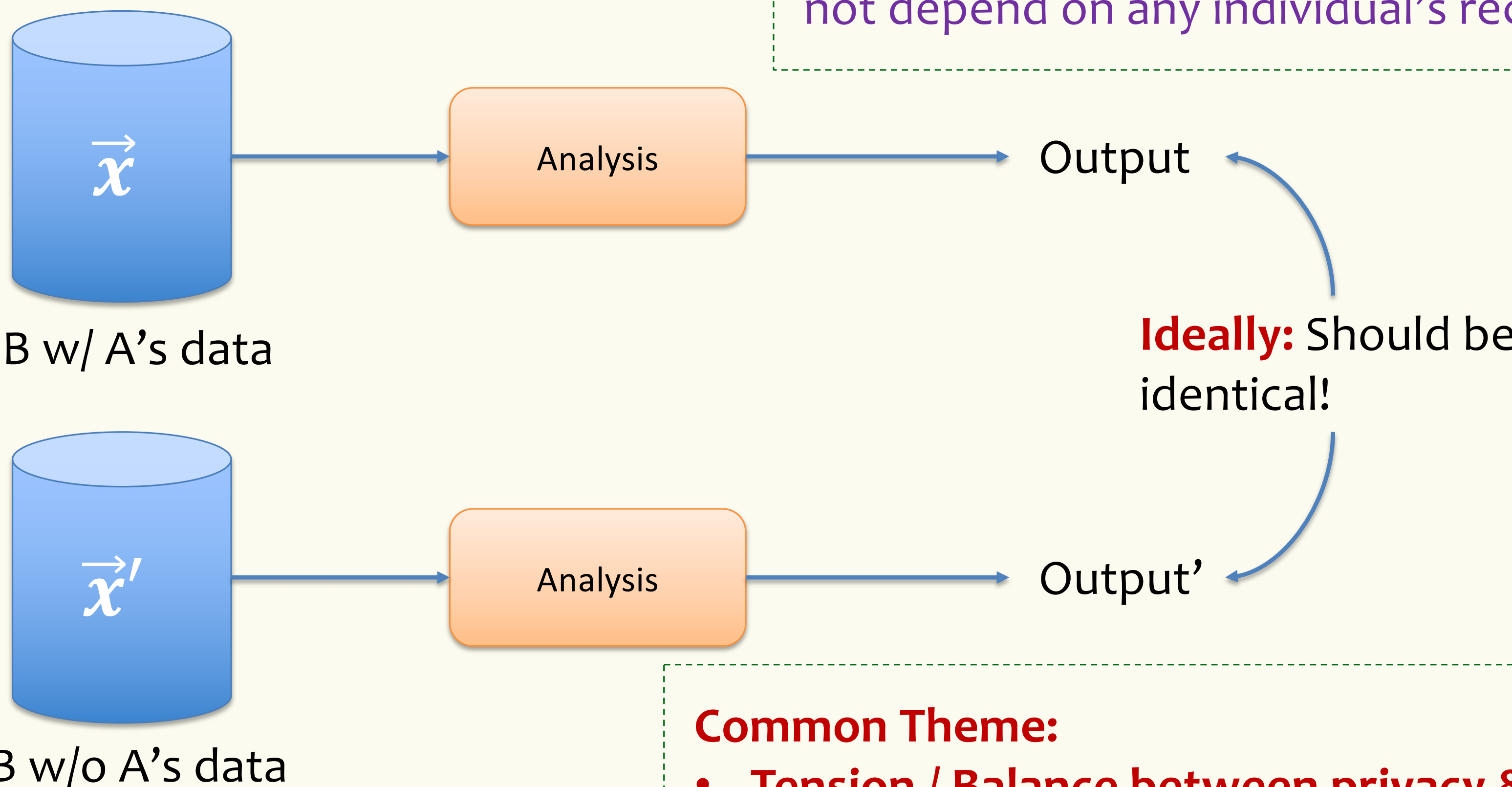
One way out? Differential Privacy

- A **formal definition** of privacy
  - Satisfied in systems deployed by Google, Uber, Apple, …
- Used by 2020 census
- Idea: *Any information-related risk to a person should not change significantly as a result of that person's information being included, or not, in the analysis.*
  - *Even with side information!*

Ideal Individual's Privacy

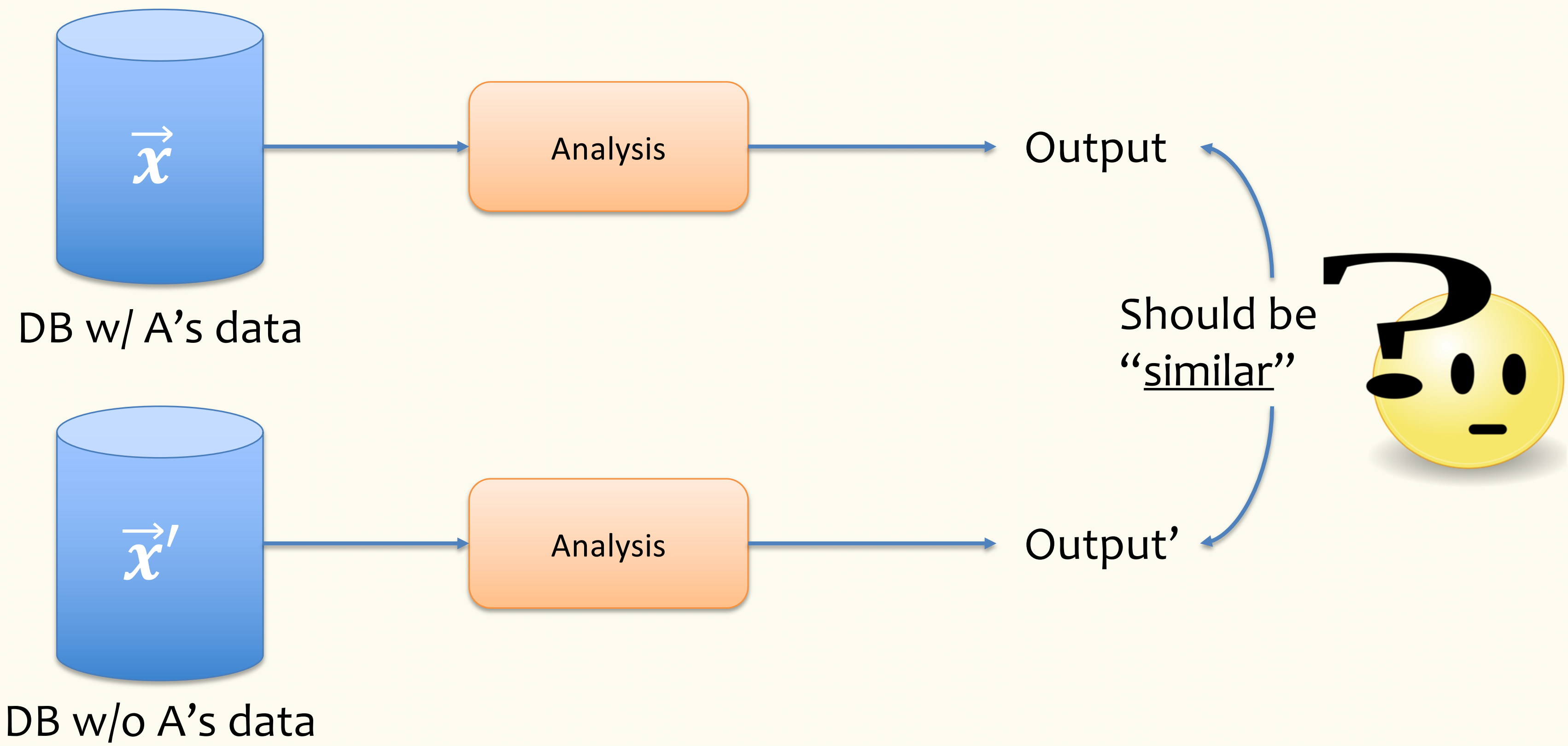
For every individual A whose record in DB

Very good for privacy.
But the output would be **useless** as it does not depend on any individual's record!



Common Theme:
- **Tension / Balance between privacy & utility**
- **Privacy is not a 0 / 1 property.**

More Realistic Privacy Goal



Setting – Formal

M = mechanism

$\vec{x}$ → M → $M(\vec{x}) \in \mathbb{R}$

w/ A's data

We say that $\vec{x}, \vec{x}'$ **differ at exactly one entry**

Here, M is <u>randomized</u>, i.e., it makes random choices

$\vec{x}'$ → M → $M(\vec{x}') \in \mathbb{R}$

w/o A's data

Setting – Mechanism

Definition. A mechanism M is ϵ-**differentially private** if for all subsets* $T \subseteq \mathbb{R}$, and for all databases $\vec{\boldsymbol{x}}, \vec{\boldsymbol{x}}'$ which differ at exactly one entry,

$$\mathbb{P}(M(\vec{\boldsymbol{x}}) \in T) \leq e^{\epsilon}\, \mathbb{P}(M(\vec{\boldsymbol{x}}') \in T)$$

Dwork, McSherry, Nissim, Smith, '06

Think: $\epsilon = \frac{1}{100}$ or $\epsilon = \frac{1}{10}$

* Can be generalized beyond output in $\mathbb{R}$

Example – Counting Queries

- DB is a vector $\vec{\boldsymbol{x}} = (x_1, \dots, x_n)$ where $x_1, \dots, x_n \in \{0,1\}$
  - $x_i = 1$ if individual i has diseases
  - $x_i = 0$ means patient does not have disease or patient data wasn't recorded.
- Query: $q(\vec{\boldsymbol{x}}) = \sum_{i=1}^{n} x_i$

Here: $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{x}}'$ differ at one entry means they differ at one single coordinate, e.g., $x_i = 1$ and $x'_i = 0$

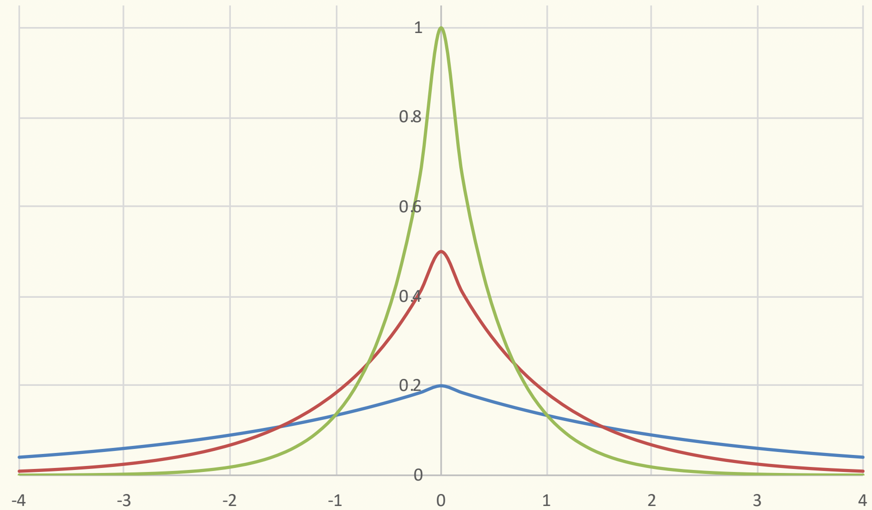
A solution – Laplacian Noise

Mechanism M taking input $\vec{\boldsymbol{x}} = (x_1, \dots, x_n)$:

- Return $M(\vec{\boldsymbol{x}}) = \sum_{i=1}^{n} x_i + Y$

"Laplacian mechanism with parameter ϵ"

Here, Y follows a **Laplace distribution** with parameter ϵ



$$f_Y(y) = \frac{\epsilon}{2} e^{-\epsilon|y|}$$

$$\mathbb{E}(Y) = 0$$

$$\mathrm{Var}(Y) = \frac{2}{\epsilon^2}$$

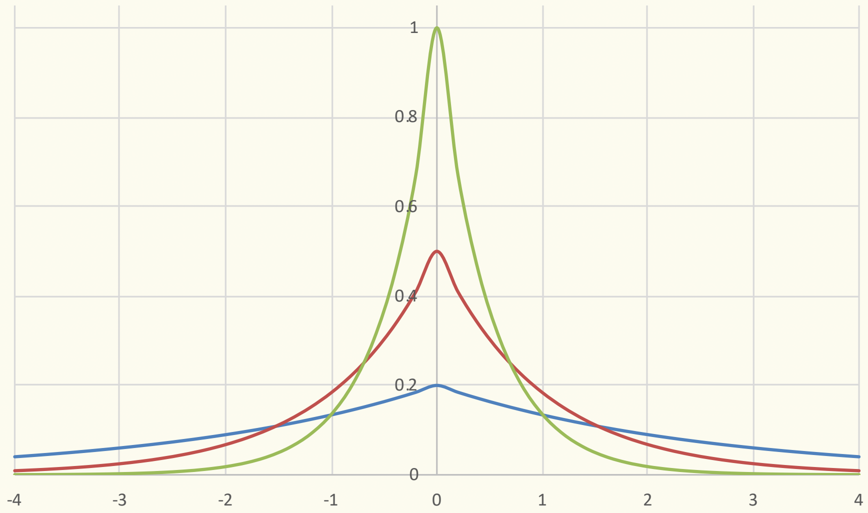
Better Solution – Laplacian Noise

Mechanism M taking input $\vec{\boldsymbol{x}} = (x_1, \ldots, x_n)$:

- Return $M(\vec{\boldsymbol{x}}) = \sum_{i=1}^{n} x_i + Y$

"Laplacian mechanism with parameter ϵ"

Here, Y follows a **Laplace distribution** with parameter ϵ



$$f_Y(y) = \frac{\epsilon}{2} e^{-\epsilon|y|}$$

Key property: For all y, Δ

$$\frac{f_Y(y)}{f_Y(y+\Delta)} \leq e^{\epsilon\Delta}$$

Laplacian Mechanism – Privacy

Theorem. The Laplacian Mechanism with parameter ϵ satisfies ϵ-differential privacy

Show: $\forall\ \vec{\boldsymbol{x}}, \vec{\boldsymbol{x}}'$ differ at one entry, $[a,b]$

$$\mathbb{P}(M(\vec{\boldsymbol{x}}) \in [a,b]) \leq e^{\epsilon}\mathbb{P}(M(\vec{\boldsymbol{x}}') \in [a,b])$$

$$\Delta = \underbrace{\sum_{i=1}^{n} x'_i}_{= s'} - \underbrace{\sum_{i=1}^{n} x_i}_{= s} \qquad |\Delta| \leq 1$$

$$\mathbb{P}(M(\vec{\boldsymbol{x}}) \in [a,b]) = \mathbb{P}(s + Y \in [a,b]) = \int_{a-s}^{b-s} f_Y(y)\mathrm{d}y = \int_a^b f_Y(y' - s)\mathrm{d}y'$$

$$= \int_a^b f_Y(y - s' + \Delta)\mathrm{d}y \leq e^{\epsilon\Delta}\int_a^b f_Y(y - s')\mathrm{d}y \leq e^{\epsilon}\int_a^b f_Y(y - s')\mathrm{d}y$$

$$\leq e^{\epsilon}\mathbb{P}(M(\vec{\boldsymbol{x}}') \in [a,b])$$

How Accurate is Laplacian Mechanism?

Let’s look at $\sum_{i=1}^{n} x_i + Y$

- $\mathbb{E}(\sum_{i=1}^{n} x_i + Y) = \sum_{i=1}^{n} x_i + \mathbb{E}(Y) = \sum_{i=1}^{n} x_i$
- $\mathrm{Var}(\sum_{i=1}^{n} x_i + Y) = \mathrm{Var}(Y) = \frac{2}{\epsilon^2}$

This is accurate enough for large enough n!

Differential Privacy – What else can we compute?

- **Statistics:** counts, mean, median, histograms, boxplots, etc.
- **Machine learning:** classification, regression, clustering, distribution learning, etc.
- **...**

Differential Privacy – Nice Properties

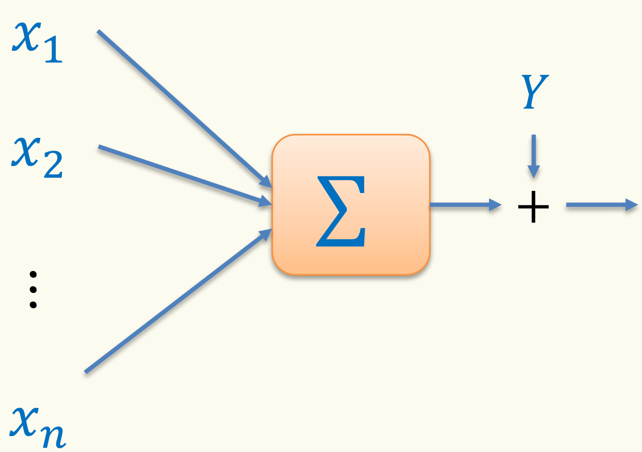
- **Group privacy:** If M is ϵ-differentially private, then for all $T \subseteq \mathbb{R}$, and <u>for all</u> databases $\vec{\boldsymbol{x}}, \vec{\boldsymbol{x}}'$ which differ at (at most) k entries,

$$\mathbb{P}(M(\vec{\boldsymbol{x}}) \in T) \leq e^{k\epsilon}\, \mathbb{P}(M(\vec{\boldsymbol{x}}') \in T)$$

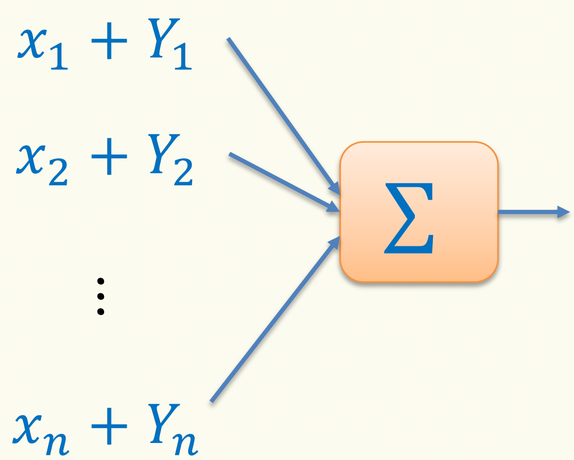
- **Composition:** If we apply two ϵ-DP mechanisms to data, combined output is 2ϵ-DP.
  - How much can we allow ϵ to grow? (So-called "privacy budget.")
- **Post-processing:** Postprocessing does not decrease privacy.

Local Differential Privacy

Laplacian Mechanism



What if we don't trust aggregator?



Solution: Add noise <u>locally</u>!

Example – Randomize Response

For a given parameter α

Mechanism M taking input $\vec{\boldsymbol{x}} = (x_1, \dots, x_n)$:

- For all $i = 1, \dots, n$:
  - $y_i = x_i$ w/ probability $\frac{1}{2} + \alpha$, and $y_i = 1 - x_i$ w/ probability $\frac{1}{2} - \alpha$.
  - $\hat{x}_i = \frac{y_i - \frac{1}{2} + \alpha}{2\alpha}$
- Return $M(\vec{\boldsymbol{x}}) = \sum_{i=1}^{n} \hat{x}_i$

S. L. Warner. Randomized response: A survey technique for eliminating evasive answer bias. Journal of the American Statistical Association, 60(309):63–69, 1965

Example – Randomize Response

For a given parameter α

Mechanism M taking input $\vec{\boldsymbol{x}} = (x_1, \dots, x_n)$:

- For all $i = 1, \dots, n$:
  - $y_i = x_i$ w/ probability $\frac{1}{2} + \alpha$, and $y_i = 1 - x_i$ w/ probability $\frac{1}{2} - \alpha$.
  - $\hat{x}_i = \frac{y_i - \frac{1}{2} + \alpha}{2\alpha}$
- Return $M(\vec{\boldsymbol{x}}) = \sum_{i=1}^{n} \hat{x}_i$

Theorem. Randomized Response with parameter α satisfies ϵ-differential privacy, if $\alpha = \frac{e^{\epsilon}-1}{e^{\epsilon}+1}$.

Fact 1. $\mathbb{E}\left(M(\vec{\boldsymbol{x}})\right) = \sum_{i=1}^{n} x_i$

Fact 2. $\mathrm{Var}\left(M(\vec{\boldsymbol{x}})\right) \approx \frac{n}{\epsilon^2}$

Differential Privacy – Challenges

- **Accuracy vs. privacy:** How do we choose ϵ?
  - Practical applications tend to err in favor of accuracy.
  - See e.g. https://arxiv.org/abs/1709.02753
- **Fairness:** Differential privacy hides contribution of small groups, by design
  - How do we avoid excluding minorities?
  - Very hard problem!

Literature

- Cynthia Dwork and Aaron Roth. *“The Algorithmic Foundations of Differential Privacy”*.
  - https://www.cis.upenn.edu/~aaroth/Papers/privacybook.pdf
- https://privacytools.seas.harvard.edu/

Epilogue

- Thank you for working hard with me and the TAs throughout this difficult time. You have shown your resilience!
- I feel fortunate to have the experience to teach you in this class.
- Enjoy a great summer and come back stronger.