CSE 312: Foundations of Computing II

Section 5: Important Discrete Distributions, More practice with r.v.s Solutions

1. Review of Independence and Consequences

(a) **Independence**: Random variable X and event E are independent iff

$$\forall x, \quad \mathbb{P}(X = x \cap E) = \mathbb{P}(X = x)\mathbb{P}(E)$$

Random variables X and Y are independent iff

$$\forall x \forall y, \quad \mathbb{P}(X = x \cap Y = y) = \mathbb{P}(X = x)\mathbb{P}(Y = y)$$

In this case, we have $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$ (the converse is not necessarily true).

(b) **i.i.d. (independent and identically distributed)**: Random variables $X_1, \ldots, X_n$ are i.i.d. (or iid) iff they are mutually independent and have the same probability mass function.

(c) **Independence of functions of a r.v.**: If X and Y are independent and $g(\cdot), h(\cdot)$ are functions mapping real numbers to real numbers, then $g(X)$ and $h(Y)$ are independent. (See if you can prove this!)

(d) **Variance of Independent Variables**: If X is independent of Y, Var $(X + Y) =$ Var $(X) +$ Var(Y). This depends on independence, whereas linearity of expectation always holds. Note that this combined with the above shows that $\forall a, b, c \in \mathbb{R}$ and if X is independent of Y, $Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y)$.

2. Review: Zoo of Discrete Random Variables

(a) **Uniform**: $X \sim$ Uniform(a, b) (Unif(a, b) for short), for integers $a \leq b$, iff X has the following probability mass function:

$$p_X(k) = \frac{1}{b - a + 1}, \quad k = a, a + 1, \ldots, b$$

$\mathbb{E}[X] = \frac{a+b}{2}$ and $Var(X) = \frac{(b-a)(b-a+2)}{12}$. This represents each integer from $[a, b]$ to be equally likely. For example, a single roll of a fair die is Uniform$(1, 6)$.

(b) **Bernoulli (or indicator)**: $X \sim$ Bernoulli(p) (Ber(p) for short) iff X has the following probability mass function:

$$p_X(k) = \begin{cases} p, & k = 1 \\ 1 - p, & k = 0 \end{cases}$$

$\mathbb{E}[X] = p$ and $Var(X) = p(1 - p)$. An example of a Bernoulli r.v. is one flip of a coin with $\mathbb{P}(\text{head}) = p$.

(c) **Binomial**: $X \sim$ Binomial(n, p) (Bin(n, p) for short) iff X is the sum of n iid Bernoulli(p) random variables. X has probability mass function

$$p_X(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad k = 0, 1, \ldots, n$$

$\mathbb{E}[X] = np$ and $Var(X) = np(1 - p)$. An example of a Binomial r.v. is the number of heads in n independent flips of a coin with $\mathbb{P}(\text{head}) = p$. Note that Bin$(1, p) \equiv$ Ber(p). As $n \to \infty$ and $p \to 0$, with $np = \lambda$, then Bin $(n, p) \to$ Poi(λ). If $X_1, \ldots, X_n$ are independent Binomial r.v.'s, where $X_i \sim$ Bin(N_i, p), then $X = X_1 + \ldots + X_n \sim$ Bin$(N_1 + \ldots + N_n, p)$.

(d) **Geometric:** $X \sim$ Geometric(p) (Geo(p) for short) iff X has the following probability mass function:

$$p_X(k) = (1 - p)^{k-1} p, \quad k = 1, 2, \ldots$$

$\mathbb{E}[X] = \frac{1}{p}$ and $Var(X) = \frac{1-p}{p^2}$. An example of a Geometric r.v. is the number of independent coin flips up to and including the first head, where $\mathbb{P}(\text{head}) = p$.

(e) **Poisson**: $X \sim$ Poisson(λ) (Poi(λ) for short) iff X has the following probability mass function:

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$$

$\mathbb{E}[X] = \lambda$ and $Var(X) = \lambda$. An example of a Poisson r.v. is the number of people born during a particular minute, where λ is the average birth rate per minute. If $X_1, \ldots, X_n$ are independent Poisson r.v.'s, where $X_i \sim$ Poi(λ_i), then $X = X_1 + \ldots + X_n \sim$ Poi($\lambda_1 + \ldots + \lambda_n$).

(f) **Negative Binomial**: $X \sim$ NegativeBinonial(r, p) (NegBin(r, p) for short) iff X is the sum of r iid Geometric(p) random variables. X has probability mass function

$$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, \quad k = r, r+1, \ldots$$

$\mathbb{E}[X] = \frac{r}{p}$ and $Var(X) = \frac{r(1-p)}{p^2}$. An example of a Negative Binomial r.v. is the number of independent coin flips up to and including the r^{th} head, where $\mathbb{P}(\text{head}) = p$. If $X_1, \ldots, X_n$ are independent Negative Binomial r.v.'s, where $X_i \sim$ NegBin(r_i, p), then $X = X_1 + \ldots + X_n \sim$ NegBin($r_1 + \ldots + r_n, p$).

(g) **Hypergeometric**: $X \sim$ HyperGeometric(N, K, n) (HypGeo(N, K, n) for short) iff X has the following probability mass function:

$$p_X(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}, \quad k = \max\{0, n+K-N\}, \ldots, \min\{K, n\}$$

$\mathbb{E}[X] = n\frac{K}{N}$. This represents the number of successes drawn, when n items are drawn from a bag with N items (K of which are successes, and $N - K$ failures) without replacement. If we did this with replacement, then this scenario would be represented as Bin $\left(n, \frac{K}{N}\right)$.

3. Pond Fishing

Suppose I am fishing in a pond with B blue fish, R red fish, and G green fish, where $B + R + G = N$. For each of the following scenarios, identify the most appropriate distribution (with parameter(s)):

(a) how many of the next 10 fish I catch are blue, if I catch and release

Solution:

Since this is the same as saying how many of my next 10 trials (fish) are a success (are blue), this is a binomial distribution. Specifically, since we are doing catch and release, the probability of a given fish being blue is $\frac{B}{N}$ and each trial is independent. Thus:

$$\text{Bin}\left(10, \frac{B}{N}\right)$$

(b) how many fish I had to catch until my first green fish, if I catch and release

Solution:

Once again, each catch is independent, so this is asking how many trials until we see a success, hence it is a geometric distribution:

$$\text{Geo}\left(\frac{G}{N}\right)$$

(c) how many red fish I catch in the next five minutes, if I catch on average r red fish per minute

Solution:

This is asking for the number of occurrences of event given an average rate, which is the definition of the Poisson distribution. Since we're looking for events in the next 5 minutes, that is our time unit, so we have to adjust the average rate to match (r per minute becomes $5r$ per 5 minutes).

$$\text{Poi}(5r)$$

(d) whether or not my next fish is blue

Solution:

This is the same as the binomial case, but it's only one trial, so it is necessarily Bernoulli.

$$\text{Ber}\left(\frac{B}{N}\right)$$

(e) (optional) how many of the next 10 fish I catch are blue, if I do not release the fish back to the pond after each catch

Solution:

We have not covered the Hypergeometric RV in class, but its definition is the number of successes in n draws (without replacement) from N items that contain K successes in total. In this case, we have 10 draws (without replacement because we do not catch and release), and out of the N fish, B are blue (a success).

$$\text{HypGeo}(N, B, 10)$$

(f) (optional) how many fish I have to catch until I catch three red fish, if I catch and release

Solution:

Negative binomial is another RV we didn't cover in class. It models the number of trials with probability of success p, until you get r successes. In this case, as before, our trials are caught fish (with replacement this time) and our success is if the fish are red, which happens with probability $\frac{R}{N}$.

$$\text{NegBin}\left(3, \frac{R}{N}\right)$$

4. Best Coach Ever!!

You are a hardworking boxer. Your coach tells you that the probability of your winning a boxing match is 0.2 independently of every other match.

(a) How many matches do you expect to fight until you win 10 times and what kind of random variable is this?

Solution:

The number of matches you have to fight until you win 10 times can be modeled by $\sum_{i=1}^{10} X_i$ where $X_i \sim \text{Geometric}(0.2)$ is the number of matches you have to fight to go from $i-1$ wins to i wins, including the match that gets you your i^{th} win, where every match has a 0.2 probability of success. Recall $\mathbb{E}[X_i] = \frac{1}{0.2} = 5$. $\mathbb{E}\left[\sum_{i=1}^{10} X_i\right] = \sum_{i=1}^{10} \mathbb{E}[X_i] = \sum_{i}^{10} \frac{1}{0.2} = 10 \cdot 5 = 50.$

(b) You only get to play 12 matches every year. To win a spot in the Annual Boxing Championship, a boxer needs to win at least 10 matches in a year. What is the probability that you will go to the Championship this year and what kind of random variable is the number of matches you win out of the 12?

Solution:

You can go to the championship if you win more than or equal to 10 times this year. Let Y be the number of matches you win out of the 12 matches. Note that $Y \sim$ Binomial$(12, 0.2)$. Since the max number you can win is 12 (there are 12 matches), we are looking for $P(10 \leq Y \leq 12)$. Thus, since Y is discrete, we are interested in

$$\mathbb{P}(Y = 10) + \mathbb{P}(Y = 11) + \mathbb{P}(Y = 12) = \sum_{i=10}^{12} \binom{12}{i} 0.2^i (1 - 0.2)^{12-i}$$

(c) Let p be your answer to part (b). How many times can you expect to go to the Championship in your 20 year career?

Solution:

The number of times you go to the championship can be modeled by $Y \sim$ Binomial$(20, p)$. So, $E[Y] = 20 \cdot p$.

5. Variance of a Product

Let X, Y, Z be independent random variables with means μ_X, μ_Y, μ_Z and variances $\sigma_X^2, \sigma_Y^2, \sigma_Z^2$, respectively. Find $Var(XY - Z)$.

Solution:

First notice that $Var(X) = \mathbb{E}\left[X^2\right] - \mathbb{E}[X]^2 \implies \mathbb{E}\left[X^2\right] = Var(X) + \mathbb{E}[X]^2 = \sigma_X^2 + \mu_X^2$, and same for Y.

$$\begin{aligned} Var(XY) &= \mathbb{E}\left[X^2Y^2\right] - \mathbb{E}[XY]^2 \text{ (by theorem in class)} \\ &= \mathbb{E}\left[X^2\right] \mathbb{E}\left[Y^2\right] - (\mathbb{E}[X]\,\mathbb{E}[Y])^2 \text{(by independence)} \\ &= \mathbb{E}\left[X^2\right] \mathbb{E}\left[Y^2\right] - \mathbb{E}[X]^2\, \mathbb{E}[Y]^2 \\ &= (\sigma_X^2 + \mu_X^2)(\sigma_Y^2 + \mu_Y^2) - \mu_X^2 \mu_Y^2 \end{aligned}$$

By independence,

$$\begin{aligned} Var(XY - Z) &= Var(XY) + Var(-Z) = Var(XY) + Var(Z) \\ &= (\sigma_X^2 + \mu_X^2)(\sigma_Y^2 + \mu_Y^2) - \mu_X^2 \mu_Y^2 + \sigma_Z^2 \end{aligned}$$

6. True or False?

Identify the following statements as true or false (true means always true). Justify your answer.

(a) For any random variable X, we have $\mathbb{E}\left[X^2\right] \geq \mathbb{E}[X]^2$.

Solution:

True, since $0 \leq Var(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$, since the squaring necessitates the result is non-negative.

(b) Let X, Y be random variables. Then, X and Y are independent if and only if $\mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$.

Solution:

False. The forward implication is true, but the reverse is not. For example, if $X \sim$ Uniform$(-1, 1)$ (equally likely to be in $\{-1, 0, 1\}$), and $Y = X^2$, we have $\mathbb{E}[X] = 0$, so $\mathbb{E}[X]\,\mathbb{E}[Y] = 0$. However, since $X = X^3$ (why?), $\mathbb{E}[XY] = \mathbb{E}\left[XX^2\right] = \mathbb{E}\left[X^3\right] = \mathbb{E}[X] = 0$, we have that $\mathbb{E}[X]\,\mathbb{E}[Y] = 0 = \mathbb{E}[XY]$. However, X and Y are not independent; indeed, $\mathbb{P}(Y = 0|X = 0) = 1 \neq \frac{1}{3} = \mathbb{P}(Y = 0)$.

(c) Let $X \sim$ Binomial(n, p) and $Y \sim$ Binomial(m, p) be independent. Then, $X + Y \sim$ Binomial$(n + m, p)$.

Solution:

True. X is the sum of n independent Bernoulli trials, and Y is the sum of m. So $X+Y$ is the sum of $n+m$ independent Bernoulli trials, so $X+Y \sim$ Binomial$(n+m,p)$.

(d) Let $X_1,...,X_{n+1}$ be independent Bernoulli(p) random variables. Then, $\mathbb{E}[\sum_{i=1}^n X_iX_{i+1}] = np^2$.

Solution:

True. Notice that X_iX_{i+1} is also Bernoulli (only takes on 0 and 1), but is 1 iff both are 1, so $X_iX_{i+1} \sim$ Bernoulli(p^2). The statement holds by linearity, since $\mathbb{E}[X_iX_{i+1}] = p^2$.

(e) Let $X_1,...,X_{n+1}$ be independent Bernoulli(p) random variables. Then, $Y = \sum_{i=1}^n X_iX_{i+1} \sim$ Binomial(n,p^2).

Solution:

False. They are all Bernoulli p^2 as determined in the previous part, but they are not independent. Indeed, $\mathbb{P}(X_1X_2 = 1|X_2X_3 = 1) = \mathbb{P}(X_1 = 1) = p \neq p^2 = \mathbb{P}(X_1X_2 = 1)$.

(f) If $X \sim$ Bernoulli(p), then $nX \sim$ Binomial(n,p).

Solution:

False. The range of X is $\{0,1\}$, so the range of nX is $\{0,n\}$. nX cannot be Bin(n,p), otherwise its range would be $\{0,1,...,n\}$.

(g) If $X \sim$ Binomial(n,p), then $\frac{X}{n} \sim$ Bernoulli(p).

Solution:

False. Again, the range of X is $\{0,1,...,n\}$, so the range of $\frac{X}{n}$ is $\{0,\frac{1}{n},\frac{2}{n},...,1\}$. Hence it cannot be Ber$(p)$, otherwise its range would be $\{0,1\}$.

(h) For any two independent random variables X,Y, we have $Var(X-Y) = Var(X) - Var(Y)$.

Solution:

False. $Var(X-Y) = Var(X+(-Y)) = Var(X) + (-1)^2Var(Y) = Var(X) + Var(Y)$.

7. Fun with Poissons

Let $X \sim Poisson(\lambda_1)$ and $Y \sim Poisson(\lambda_2)$, and X and Y are independent.

(a) [This was done in class.] Show that $X+Y \sim Poisson(\lambda_1+\lambda_2)$

Solution:

To show that a random variable is distributed according to a particular distribution, we must show that they have the same PMF. Thus, we are trying to show that $P(X+Y=n) = e^{-(\lambda_1+\lambda_2)}\frac{(\lambda_1+\lambda_2)^n}{n!}$

$$\begin{aligned}
P(X+Y=n) &= \sum_{k=0}^{n} P(X=k \cap Y=n-k) \\
&= \sum_{k=0}^{n} P(X=k)P(Y=n-k) && \text{[X and Y are independent]} \\
&= \sum_{k=0}^{n} e^{-\lambda_1}\frac{\lambda_1^k}{k!}\, e^{-\lambda_2}\frac{\lambda_2^{n-k}}{(n-k)!} \\
&= e^{-(\lambda_1+\lambda_2)}\sum_{k=0}^{n} \frac{\lambda_1^k}{k!}\,\frac{\lambda_2^{n-k}}{(n-k)!} \\
&= e^{-(\lambda_1+\lambda_2)}\sum_{k=0}^{n} \frac{1}{k!(n-k)!}\,\lambda_1^k\lambda_2^{n-k} \\
&= \frac{e^{-(\lambda_1+\lambda_2)}}{n!}\sum_{k=0}^{n} \frac{n!}{k!(n-k)!}\,\lambda_1^k\lambda_2^{n-k} \\
&= \frac{e^{-(\lambda_1+\lambda_2)}}{n!}\sum_{k=0}^{n} \binom{n}{k}\,\lambda_1^k\lambda_2^{n-k} \\
&= \frac{e^{-(\lambda_1+\lambda_2)}}{n!}\,(\lambda_1+\lambda_2)^n && \text{[Binomial Theorem]}
\end{aligned}$$

(b) Show that $P(X=k \mid X+Y=n) = P(W=k)$ where $W \sim Bin(n, \frac{\lambda_1}{\lambda_1+\lambda_2})$

Solution:

$$\begin{aligned}
P(X=k \mid X+Y=n) &= \frac{P(X=k \cap X+Y=n)}{P(X+Y=n)} \\
&= \frac{P(X=k \cap Y=n-k)}{P(X+Y=n)} \\
&= \frac{P(X=k)P(Y=n-k)}{P(X+Y=n)} && \text{[X and Y are independent]} \\
&= \frac{e^{-\lambda_1}\frac{\lambda_1^k}{k!} \cdot e^{-\lambda_2}\frac{\lambda_2^{n-k}}{(n-k)!}}{e^{-(\lambda_1+\lambda_2)}\frac{(\lambda_1+\lambda_2)^n}{n!}} \\
&= \frac{\frac{\lambda_1^k}{k!} \cdot \frac{\lambda_2^{n-k}}{(n-k)!}}{\frac{(\lambda_1+\lambda_2)^n}{n!}} \\
&= \frac{n!}{k!(n-k)!} \cdot \frac{\lambda_1^k \lambda_2^{n-k}}{(\lambda_1+\lambda_2)^n} \\
&= \binom{n}{k} \frac{\lambda_1^k\ \lambda_2^{n-k}}{(\lambda_1+\lambda_2)^k\ (\lambda_1+\lambda_2)^{n-k}} \\
&= \binom{n}{k} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k} \\
&= \binom{n}{k} p^k\ (1-p)^{n-k}\ \text{, where } p = \frac{\lambda_1}{\lambda_1+\lambda_2}
\end{aligned}$$

8. Memorylessness

We say that a random variable X is memoryless if $\mathbb{P}(X > k+i \mid X > k) = \mathbb{P}(X > i)$ for all non-negative integers k and i. The idea is that X does not *remember* its history. Let $X \sim Geo(p)$. Show that X is memoryless.

Solution:

Let's note that if $X \sim Geo(p)$, then $\mathbb{P}(X > k) = \mathbb{P}(\text{no successes in the first } k \text{ trials}) = (1-p)^k$.

$$\begin{aligned}
\mathbb{P}(X > k+i \mid X > k) &= \frac{\mathbb{P}(X > k \mid X > k+i)\ \mathbb{P}(X > k+i)}{\mathbb{P}(X > k)} && \text{[Bayes Theorem]} \\
&= \frac{\mathbb{P}(X > k+i)}{\mathbb{P}(X > k)} && [\mathbb{P}(X > k \mid X > k+i) = 1] \\
&= \frac{(1-p)^{k+i}}{(1-p)^k} && [\mathbb{P}(X > k) = (1-p)^k] \\
&= (1-p)^i \\
&= \mathbb{P}(X > i)
\end{aligned}$$

9. Review of Main Concepts for Continuous Random Variables

(a) **Cumulative Distribution Function (cdf)**: For any random variable (discrete or continuous) X, the cumulative distribution function is defined as $F_X(x) = \mathbb{P}(X \le x)$. Notice that this function must be monotonically nondecreasing: if $x < y$ then $F_X(x) \le F_X(y)$, because $\mathbb{P}(X \le x) \le \mathbb{P}(X \le y)$. Also notice that since probabilities are between 0 and 1, that $0 \le F_X(x) \le 1$ for all x, with $\lim_{x\to-\infty} F_X(x) = 0$ and $\lim_{x\to+\infty} F_X(x) = 1$.

(b) **Continuous Random Variable**: A continuous random variable X is one for which its cumulative distribution function $F_X(x) : \mathbb{R} \to \mathbb{R}$ is continuous everywhere. A continuous random variable has an uncountably infinite number of values.

(c) **Probability Density Function (pdf or density)**: Let X be a continuous random variable. Then the probability density function $f_X(x) : \mathbb{R} \to \mathbb{R}$ of X is defined as $f_X(x) = \frac{d}{dx} F_X(x)$. Turning this around, it means that $F_X(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\, dt$. From this, it follows that $\mathbb{P}(a \le X \le b) = F_X(b) - F_X(a) = \int_a^b f_X(x) dx$ and that $\int_{-\infty}^{\infty} f_X(x) dx = 1$. From the fact that $F_X(x)$ is monotonically nondecreasing it follows that $f_X(x) \ge 0$ for every real number x.

If X is a continuous random variable, note that in general $f_X(a) \neq \mathbb{P}(X = a)$, since $\mathbb{P}(X = a) = F_X(a) - F_X(a) = 0$ for all a. However, the probability that X is close to a is proportional to $f_X(a)$: for small δ, $\mathbb{P}\left(a - \frac{\delta}{2} < X < a + \frac{\delta}{2}\right) \approx \delta f_X(a)$.

(d) **Discrete to Continuous:**

	Discrete	**Continuous**
PMF/PDF	$p_X(x) = \mathbb{P}(X = x)$	$f_X(x) \neq \mathbb{P}(X = x) = 0$
CDF	$F_X(x) = \sum_{t \le x} p_X(t)$	$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$
Normalization	$\sum_x p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
Expectation	$\mathbb{E}[X] = \sum_x x \cdot p_X(x)$	$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$
LOTUS	$\mathbb{E}[g(X)] = \sum_x g(x) p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$

10. Continuous r.v. example

Suppose that X is a random variable with pdf

$$f_X(x) = \begin{cases} 2C(2x - x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

where C is an appropriately chosen constant.

(a) What must the constant C be for this to be a valid pdf?

Solution:

For $f_X(x)$ to be a valid PDF, the area under the graph must be 1. Computing the area under the graph as a function of C gives us

$$\int_{-\infty}^{\infty} f_X(x) dx = \int_0^2 2C(2x - x^2) dx = 2C\left(x^2 - \frac{1}{3}x^3 \Big|_0^2\right) = 2C\frac{4}{3} = \frac{8}{3}C$$

Setting this equation equal to 1, and solving for C gives use $C = \frac{3}{8}$.

(b) What is $\mathbb{P}(X > 1)$?

Solution:

The $\mathbb{P}(X > 1) = \int_1^{\infty} f_X(t) dt$. Using our value for C that we found in the previous part we can compute this integral as follows:

$$\int_1^{\infty} f_X(t) dt = \int_1^2 \frac{6}{8}\left(2x - x^2\right) dt = \frac{6}{8}\left(x^2 - \frac{1}{3}x^3\right)\Big|_1^2 = \frac{1}{2}$$

Alternatively, $\mathbb{P}(X > 1) = 1 - \mathbb{P}(X \le 1) = 1 - F_X(1) = 1 - \int_{-\infty}^{1} f_X(t) dt$. Using our value for C that we found in the previous part we can compute this integral as follows:

$$\int_{-\infty}^{1} f_X(t) dt = \int_0^1 \frac{6}{8}\left(2x - x^2\right) dt = \frac{6}{8}\left(x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{2}$$

Plugging this value into our initial equation gives $P(X > 1) = 1 - \frac{1}{2} = \frac{1}{2}$.

(c) What is $E(X)$?

Solution:

By the definition of expectation for continuous R.V.s

$$E[X] = \int_{-\infty}^{\infty} x\ f_X(x)\ dx = \int_0^2 x\ \frac{6}{8}(2x - x^2)dx = \frac{6}{8}\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^2 = \frac{6}{8}\left(\frac{2(2)^3}{3} - \frac{2^4}{4}\right) = 1$$