CSE 312

Foundations of Computing II

Lecture 20: Continuity Correction & Distinct Elements

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Rachel Lin, Hunter Schafer

Slide Credit: Based on Stefano Tessaro's slides for 312 19au
incorporating ideas from Alex Tsun's and Anna Karlin's slides for 312 20su and 20au

Application: Distinct Elements (code this in Pset 6)

Data mining – Stream Model

- In many data mining situations, the data is not known ahead of time.
  Examples: Google queries, Twitter or Facebook status updates
  Youtube video views
- In some ways, best to think of the data as an infinite stream that is non-stationary (distribution changes over time)
- Input elements (e.g. Google queries) enter/arrive one at a time.
  We cannot possibly store the stream.

Question: How do we make critical calculations about the data stream using a limited amount of memory?

Problem Setup

- Input: sequence of N elements $x_1, x_2, \ldots, x_N$ from a known universe U (e.g., 8-byte integers).
- Goal: perform a computation on the input, in a single left to right pass where
  - Elements processed in real time
  - Can't store the full data. => use minimal amount of storage while maintaining working "summary"

What can we compute?

32, 12, 14, 32, 7, 12, 32, 7, 32, 12, 4

- Some functions are easy:
  - Min
  - Max
  - Sum
  - Average

Today: Counting distinct elements

32, 12, 14, 32, 7, 12, 32, 7, 32, 12, 4

Application:

You are the content manager at YouTube, and you are trying to figure out the **distinct** view count for a video. How do we do that?

Note: A person can view their favorite videos several times, but they only count as 1 **distinct** view!

Other applications

- IP packet streams: How many distinct IP addresses or IP flows (source+destination IP, port, protocol)
  - * Anomaly detection, traffic monitoring
- Search: How many distinct search queries on Google on a certain topic yesterday
- Web services: how many distinct users (cookies) searched/browsed a certain term/item
  - * Advertising, marketing trends, etc.

Counting distinct elements

32, 12, 14, 32, 7, 12, 32, 7, 32, 12, 4

N = # of IDs in the stream = 11, m = # of distinct IDs in the stream = 5

Want to compute number of **distinct** IDs in the stream.

- *Naïve solution: As the data stream comes in, store all distinct IDs in a hash table.*
- *Space requirement O(m) , where m is the number of distinct IDs*

- *Consider the number of users of youtube, and the number videos on youtube. This is not feasible.*

Counting distinct elements

32, 12, 14, 32, 7, 12, 32, 7, 32, 12, 4

Want to compute number of **distinct** IDs in the stream.

- *How to do this without storing all the elements?*

Yet another super cool application of probability

Counting distinct elements

32, 12, 14, 32, 7, 12, 32, 7, 32, 12, 4

$\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3}, \boldsymbol{y_1}, y_4, \boldsymbol{y_2}, \boldsymbol{y_1}, \boldsymbol{y_4}, \boldsymbol{y_1}, y_2, y_5$

Hash function $h: U \to [0,1]$
Assumption: For distinct values in U, the function maps to iid (independent and identically distributed) Unif(0,1) random numbers.

Important: if you were to feed in two equivalent elements, the function returns the **same** number.

- So m distinct elements → m iid uniform y_i's

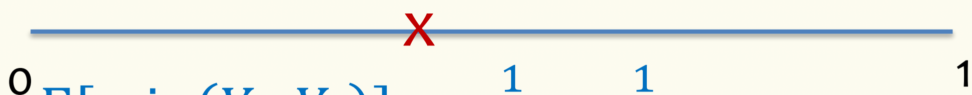
Min of IID Uniforms

If $Y_1, \cdots, Y_m$ are iid Unif(0,1), where do we expect the points to end up?

In general, $\mathrm{E}[\min(Y_1, \cdots, Y_m)] = \frac{1}{m+1}$

$$\mathrm{E}[\min(Y_1)] = \frac{1}{1+1} = \frac{1}{2}$$

$m = 1$

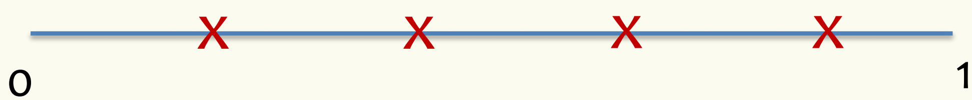


$$\mathrm{E}[\min(Y_1, Y_2)] = \frac{1}{2+1} = \frac{1}{3}$$

$m = 2$

$$\mathrm{E}[\min(Y_1, \cdots, Y_4)] = \frac{1}{4+1} = \frac{1}{5}$$

$m = 4$

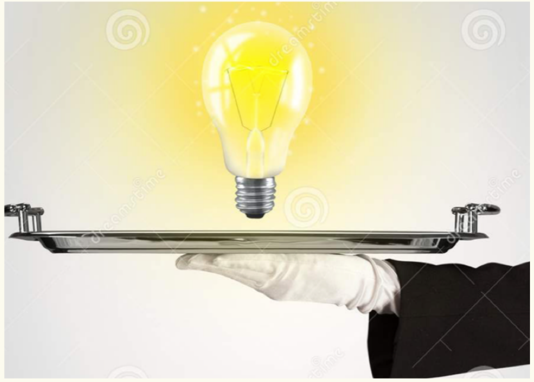


A super duper clever idea

If $Y_1, \cdots, Y_n$ are iid Unif(0,1), where do we expect the points to end up?

In general, $\mathrm{E}[\min(Y_1, \cdots, Y_m)] = \frac{1}{m+1}$

Idea: $\mathrm{m} = \frac{1}{\mathrm{E}[\min(Y_1, \cdots, Y_m)]} - 1$



Let's keep track of the value val of min of hash values, and estimate m as Round $\left(\frac{1}{val} - 1\right)$

The Distinct Elements Algorithm

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
    val $\leftarrow \infty$
function UPDATE(x)
    val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
    return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▷ Loop through all stream elements
    update(x_i) ▷ Update our single float variable
return estimate() ▷ An estimate for n, the number of distinct elements.

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes:

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow \min\{\text{val}, \text{hash(x)}\}$
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
  update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = infty

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
  update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = infty

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▷ Loop through all stream elements
  update(x_i) ▷ Update our single float variable
return estimate() ▷ An estimate for n, the number of distinct elements.

val = 0.51

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
  update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = 0.26

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26, 0.79,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▷ Loop through all stream elements
  update(x_i) ▷ Update our single float variable
return estimate() ▷ An estimate for n, the number of distinct elements.

val = 0.26

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26, 0.79, 0.26,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▷ Loop through all stream elements
  update(x_i) ▷ Update our single float variable
return estimate() ▷ An estimate for n, the number of distinct elements.

val = 0.26

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26, 0.79, 0.26, 0.79,

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▷ Loop through all stream elements
  update(x_i) ▷ Update our single float variable
return estimate() ▷ An estimate for n, the number of distinct elements.

val = 0.26

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26, 0.79, 0.26, 0.79, 0.79

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
    val $\leftarrow \infty$
function UPDATE(x)
    val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
    return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
    update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = 0.26

Distinct Elements Example

Stream: 13, 25, 19, 25, 19, 19

Hashes: 0.51, 0.26, 0.79, 0.26, 0.79, 0.79

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
    val $\leftarrow \infty$
function UPDATE(x)
    val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
    return round $\left(\frac{1}{\text{val}} - 1\right)$
for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
    update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = 0.26

Return

round(1/0.26 - 1) =

round(2.846) = 3

Diy: Distinct Elements Example II

Stream: 11, 34, 89, 11, 89, 23

Hashes: 0.5, 0.21, 0.94, 0.5, 0.94, 0.1

Algorithm 2 Distinct Elements Operations

function INITIALIZE()
  val $\leftarrow \infty$
function UPDATE(x)
  val $\leftarrow$ min {val, hash(x)}
function ESTIMATE()
  return round $\left(\frac{1}{\text{val}} - 1\right)$

for $i = 1, \ldots, N$: **do** ▹ Loop through all stream elements
  update(x_i) ▹ Update our single float variable
return estimate() ▹ An estimate for n, the number of distinct elements.

val = 0.1

Return= 9

Problem

$$\text{val} = \min(Y_1, \cdots, Y_m)$$

$$\mathrm{E}[val] = \frac{1}{m+1}$$

Algorithm:

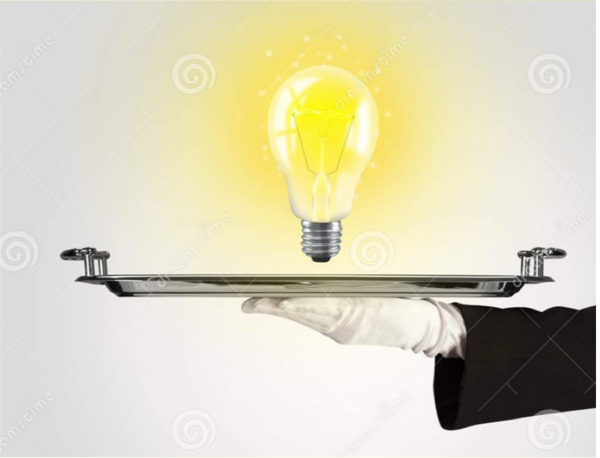
Track $val = \min\big(h(X_1), \cdots, h(X_N)\big) = \min(Y_1, \cdots, Y_m)$

estimate m = 1/val -1

But, val is not E[val]! How far is val from E[val]?

$$\mathrm{Var}[val] \approx \frac{1}{(m+1)^2}$$

How can we reduce the variance?



Idea: Repetition to reduce variance!

Use k **independent** hash functions $h^1, h^2, \cdots h^k$

Keep track of k independent min hash values

$$val^1 = \min\left(h^1(x_1), \cdots, h^1(x_N)\right) = \min(Y_1^1, \cdots, Y_m^1)$$

$$val^2 = \min\left(h^2(x_1), \cdots, h^2(x_N)\right) = \min(Y_1^2, \cdots, Y_m^2)$$

$$\dots \dots$$

$$val^k = \min\left(h^k(x_1), \cdots, h^k(x_N)\right) = \min(Y_1^k, \cdots, Y_m^k)$$

$val = \frac{1}{k}\Sigma_i val_i$, Estimate $m = \frac{1}{val} - 1$