$$Var(\hat{\theta}_1) = Var\left(\frac{1}{n}\sum_{i=1}^{n}(x_i)\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(x_i)$$
$$= \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{1}{n^2}\cdot n\sigma^2 = \frac{\sigma^2}{n}$$

$\hat{\theta}_1 \sim N(\mu, \sigma^2/n)$

$\frac{\hat{\theta}_1 - \mu}{\sqrt{\sigma^2/n}} \sim N(0,1)$

$$P\left(-z < \frac{\hat{\theta}_1 - \mu}{\sqrt{\sigma^2/n}} < +z\right) = \Phi(z) - \Phi(-z) = 2\Phi(z) - 1$$

mult by -1: $$P\left(-z < \frac{\mu - \hat{\theta}_1}{\sqrt{\sigma^2/n}} < +z\right) = 2\Phi(z) - 1$$

$$P\left(\hat{\theta}_1 - z\sqrt{\frac{\sigma^2}{n}} < \mu < \hat{\theta}_1 + z\sqrt{\sigma^2/n}\right) = 2\Phi(z) - 1 = 0.95$$

if we choose $\Delta = z\sqrt{\sigma^2/n}$

$2\Phi(z) - 1 = 0.95$

$\Phi(z) = 0.975$

$z \approx 1.96$

$$P\left(\mu \in \left[\hat{\theta}_1 - z\sqrt{\frac{\sigma^2}{n}}, \hat{\theta}_1 + z\sqrt{\frac{\sigma^2}{n}}\right)\right.$$

$$= P\left(\mu \in \left[\hat{\theta}_1 - 1.96\sqrt{\frac{\sigma^2}{n}}, \hat{\theta}_1 + 1.96\sqrt{\frac{\sigma^2}{n}}\right]\right) = 0.95$$

What if σ^2 is unknown?
Could use $\hat{\theta}_2' = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2$ as an estimator for σ^2.
If n is large, then $\frac{\hat{\theta}_1 - \mu}{\sqrt{\hat{\theta}_2'/n}}$ is approximately normal and $\Delta = 1.96\sqrt{\frac{\hat{\theta}_2'}{n}}$ gives an approximate 95% confidence interval.