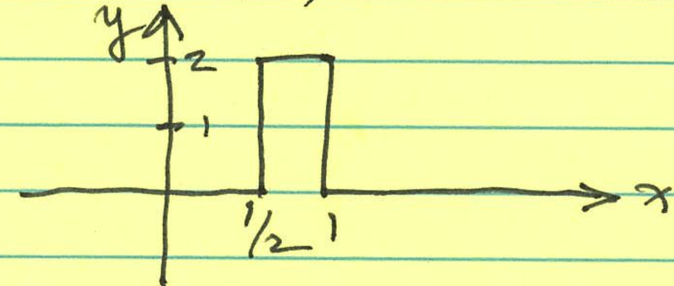
Uniform

$X \sim Uni(\alpha, \beta)$ for $\alpha < \beta$

$$f_X(x) = \begin{cases} \frac{1}{\beta - \alpha} & , \text{ if } x \in [\alpha, \beta] \\ 0 & , \text{ otherwise} \end{cases}$$

Ex: $X \sim Uni(1/2, 1)$



$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_{1/2}^{1} x \frac{1}{\beta - \alpha}\, dx$$

$$= \frac{1}{\beta - \alpha} \int_{1/2}^{1} x\, dx = \frac{1}{2(\beta - \alpha)} x^2 \Big|_{1/2}^{1}$$

In general for $X \sim Uni(\alpha, \beta)$

$$E[X] = \int_{\alpha}^{\beta} x \frac{1}{\beta - \alpha}\, dx = \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} x\, dx$$

$$= \frac{1}{2(\beta - \alpha)} x^2 \Big|_{\alpha}^{\beta} = \frac{\beta^2 - \alpha^2}{2(\beta - \alpha)} = \frac{(\beta - \alpha)(\beta + \alpha)}{2(\beta - \alpha)}$$

$$= \frac{1}{2}(\alpha + \beta)$$

$Var(X)$ can be calculated from $E[X^2]$ similarly.

Exponential: used to model the time until the next event, where events happen independently at ~~a rate of~~ an average rate of λ per time ~~interv~~ unit.

(Same setup as Poisson, which was number of events in a given time unit.

Radioactive decay: time to next α particle

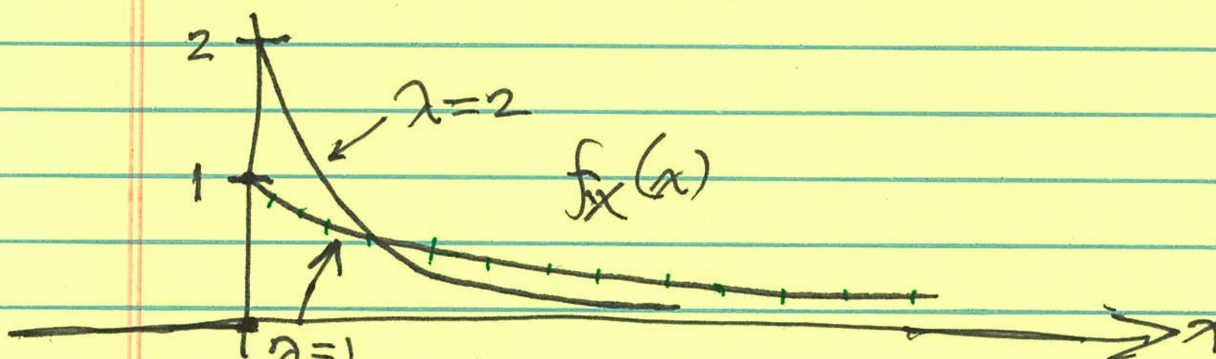
Packets: time until next packet arrives at a server

$X \sim Exp(\lambda)$

For any $t \geq 0$, $P(X \geq t) = e^{-\lambda t}$

$$F_X(x) = P(X \leq t) = 1 - P(X \geq t) = 1 - e^{-\lambda t}$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



$$E[X] = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$

Memorylessness ~~pof~~ property:

$$P(X > s+t \mid X > s) = P(X > t),$$

for any $s, t > 0$.

Exponential is the continuous analogy of the geometric distribution.

If you think of each coin flip occurring at the tick of a clock, then the number of flips until the first head is the ~~n~~ time until the first head, measured discretely.