Some important discrete random variables. We've seen one: geometric r.v.

**Defn:** $X$ is uniform on $[a, b]$, denoted $X \sim \text{Unif}(a, b)$, if $X$ is equally likely to be any integer in $[a, b]$.

- $P(X = i) = \frac{1}{b - a + 1}$ for all $i \in \{a, a+1, \ldots, b\}$.
- $E[X] = \frac{1}{2} (a + b)$
- $\text{Var}(X) = \frac{1}{12} (b - a)(b - a + 2)$.

**Ex:** One roll of a fair 6-sided die is in $\text{Unif}(1, 6)$

- $E[X] = \frac{7}{2}$, $\text{Var}(X) = \frac{35}{12}$

**Defn:** A Bernoulli r.v. $X \sim \text{Ber}(p)$ is an indicator random variable $X$ with

- $P(X = 1) = p$, $P(X = 0) = 1 - p$.
- $E[X] = 1 \cdot p + 0 \cdot (1 - p) = p$
- $E[X^2] = E[X] = p$
- $\text{Var}(X) = E[X^2] - (E[X])^2 = p - p^2 = p(1 - p)$.

**Ex:** One coin flip.

**Defn:** A binomial r.v. $X \sim \text{Bin}(n, p)$ is the sum of $n$ independent $\text{Ber}(p)$ r.v.'s $X_1, X_2, \ldots, X_n$.

- **Ex:** # heads in $n$ independent flips of a coin that has $P(\text{heads}) = p$

- $P(X = i) = \binom{n}{i} p^i (1 - p)^{n-i}$
- $E[X] = np$
- $\sum_{i=1}^{\infty} P(X_i) = \text{Var}(X) = \text{Var}(\sum_{i=1}^{\infty} X_i) = \sum_{i=1}^{\infty} \text{Var}(X_i)$, because $X_i$'s are in
\[
\text{Var}(x) = \sum_{i=1}^{m} \text{Var}(x_i) = \sum_{i=1}^{m} p(1-p) = np(1-p).
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