Some important discrete random variables.
We've seen one: geometric r.v.

Defn: $X$ is uniform on $[a,b]$, denoted $X \sim \text{Unif}(a,b)$, if $X$ is equally likely to be any integer in $[a,b]$.

$p_X(i) = \frac{1}{b-a+1}$, for all $i \in \{a, a+1, \dots, b\}$.

$E[X] = \frac{1}{2}(a+b)$

$Var(X) = \frac{1}{12}(b-a)(b-a+2)$.

Ex: One roll of a fair 6-sided die is in Unif(1,6).
$E[X] = 7/2$, $Var(X) = 35/12$

Defn: A Bernoulli r.v. $X \sim Ber(p)$ is an indicator random variable $X$ with
$P(X=1) = p$, $P(X=0) = 1-p$.

$E[X] = 1 \cdot p + 0(1-p) = p$

$E[X^2] = 1^2 \cdot p + 0^2(1-p) = p$

$Var(X) = E[X^2] - (E[X])^2 = p - p^2 = p(1-p)$.

Ex: One coin flip.

Defn: A binomial r.v. $X \sim Bin(n,p)$ is the sum of $n$ independent $Ber(p)$ r.v.'s $X_1, X_2, \dots, X_n$.

Ex: # heads in $n$ independent flips of a coin that has $P(\text{heads}) = p$.

$p_X(i) = \binom{n}{i} p^i (1-p)^{n-i}$

$E[X] = np$

$X = \sum_{i=1}^{n} X_i$ where $X_i \sim Ber(p)$ and all $n$ are independent.

$Var(X) = Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i)$, because $X_i$'s are in

$$\mathrm{Var}(X) = \sum_{i=1}^{n} \mathrm{Var}(X_i) = \sum_{i=1}^{n} p(1-p) = np(1-p).$$