

The game is a random walk on that line.

Let E_i be the event that A wins the game starting with \$ i . $P(E_i) = ?$

Condition on outcome of first flip and use LTP.

$$p_i = P(E_i) = P(E_i | H)P(H) + P(E_i | T)P(T)$$

$$= \frac{1}{2}(p_{i-1} + p_{i+1}) \text{ because } P(E_i | H) = \frac{i}{2} P(E_{i-1})$$

$$2p_i = p_{i-1} + p_{i+1}$$

$$\text{and } P(E_i | T) = P(E_{i+1})$$

$$p_i - p_{i-1} = p_{i+1} - p_i$$

$$i=1: p_2 - p_1 = p_1 - p_0 = p_1$$

$$p_2 = 2p_1$$

$$p_3 = 3p_1$$

$$p_i = ip_1 \text{ for } 0 \leq i \leq N$$

$$1 = p_N = Np_1$$

$$p_1 = 1/N$$

$$p_i = ip_1 = i/N$$