Combinations: order does not matter.

Ex. Your wizard can carry any 3 objects from: (1) sword, (2) knife, (3) staff, (4) ring, (5) laptop. How many different combinations of 3 are there?

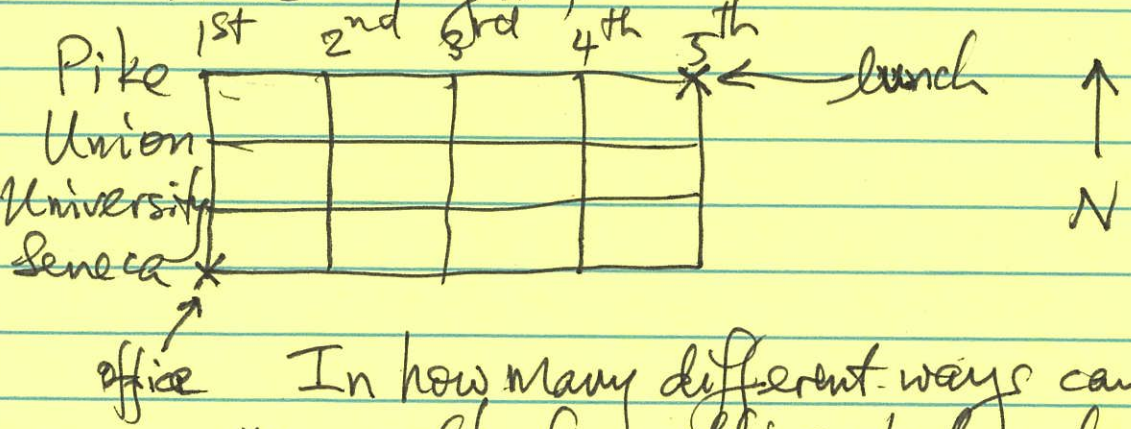
$$\frac{P(5,3)}{3!} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$$

More generally, the number of sets of k items out of n is

$$\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{k!(n-k)!}$$

"n choose k", "binomial coefficient"

($P(n,k)$: number of ordered sequences of k items out of n)



In how many different ways can you walk from office to lunch, always going either N or E at each intersection?

$$\binom{7}{3} = \frac{7!}{3!4!}$$

— N N — — N —

Ex: Assume ♠J is the face-up trump. How many starting Schnapsen hands have ≥ 1 trump?

$\binom{4}{1}\binom{18}{4} = 12{,}240$ Wrong! overcounts.

↑ #ways to deal a trump ↑ #ways to deal 4 more cards

Exactly 2 trumps: $\binom{4}{2}\binom{15}{3}$

$\binom{4}{1}\binom{18}{4}$

→ ♠Q ♠K ___

→ ♠K ♠Q ___

Correct:

How many hands have 0 trumps? $\binom{15}{5}$

How many hands in total? $\binom{19}{5}$

How many have ≥ 1 trump? $\binom{19}{5} - \binom{15}{5} = \frac{19!}{5!14!} - \frac{15!}{5!10!}$

$$= \frac{19\cdot18\cdot17\cdot16\cdot15}{5\cdot4\cdot3\cdot2\cdot1} - \frac{15\cdot14\cdot13\cdot12\cdot11}{5\cdot4\cdot3\cdot2\cdot1} = 8625$$

Another possible solution:

$$\cancel{\binom{4}{0}\binom{15}{5}} + \binom{4}{1}\binom{15}{4} + \binom{4}{2}\binom{15}{3} + \binom{4}{3}\binom{15}{2} + \binom{4}{4}\binom{15}{1}$$

Method of complements.

Identities

$\binom{n}{k} = \binom{n}{n-k}$, for $0 \le k \le n$

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$, for $1 \le k \le n-1$

Binomial Theorem: $(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$